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MEASUREMENT OF THE NEED FOR TRANSPORTING PUPILS

BASIS FOR STATE EQUALIZATION OF TRANSPORTATION COSTS

BY

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R. B.

CONTENTS

CHAPTER PAGE

I. Conditions That Give Rise to the Problem 1

Equalization of Educational Opportunity
A Minimum Program of Education
Pioneer Study of Measurement of Educational Need
Needed Supplements to this Study

II. Selection of Criterion or Independent Variable 8

Transportation Rules for New Jersey
Pupils Transported and Functions of Density
Pupils Transported and Functions of Density, School Buildings, Area
Weighted Pupils Transported and Functions of Density of School Population

III. Curve Fitting and Selection of a Curve 20

Straight Lines
Power Functions
Exponential Functions
Generalized Coefficient of Correlation

IV. Study of Factors Causing Departure from Curve 33

Types of School Buildings
Pupils Transported and Grade Location

V. Application of Curve to New Jersey Counties 42

Distribution of State Aid by Several Plans
Simplification of Curve by Use of Table

VI. Summary 52

Appendix 57
Tables 57
Bibliography 60

TABLES

TABLE		PAG
I.	Relation Between Functions of Density and Percentage of A.D.A. Transported, 1923	1
II.	Derivation of Variables X and Y for School Year of 1924	2
III.	Relation of Types of School Buildings and Residuals, New Jersey, 1924	3
IV.	Relation Between Residuals and Grade Location of Transported Pupils for 1925–1926	3
V.	Application of Transportation Index for 1924	4
VI.	State Aid Based on Transportation Index for 1924	4
VII.	Table Showing $\bar{Y}$ value for Density of School Population	4
VIII.	Derivation of Variables X and Y for 1921	5
IX.	Derivation of Variables X and Y for 1922	5
X.	Derivation of Variables X and Y for 1925	5

PLATES

PLATE PAGE

I. Graphs Showing Relation Between Percentage of Pupils Transported and Functions of the Density 13

II. Graphs Showing Relation Between Percentage of A.D.A. Transported and Indices 15

III. Graph Showing Relation Between Density of School Population and Transportation in New Jersey Counties for 1923 17

IV. Graph Showing Relation Between Log of Density of School Population and Transportation Load in New Jersey Counties, 1924 ... 23

V. Fitting of Curve, Type $Y = aX^2 + bX + c$ to Data of 1924 25

VI. Fitting of Curve, Type $Y = aX^n$ to Data of 1924 27

VII. Graphical Representation that Log Y = Log $b + a$ (log e) X is Equation of Straight Line 1924 29

VIII. Fitting of Curve, Type $Y = be^{aX}$ to Data of 1924 31

IX. Representation of Causes of Departures from Curve 37

MEASUREMENT OF THE NEED FOR TRANSPORTING PUPILS

MEASUREMENT OF THE NEED FOR TRANSPORTING PUPILS

BASIS FOR STATE EQUALIZATION OF TRANSPORTATION COSTS

CHAPTER I

CONDITIONS THAT GIVE RISE TO THE PROBLEM

Any discussion to-day of the state's responsibility with respect to public education is certain to involve the oft-repeated expression "the equalization of educational opportunity." In fact this expression is used so often that it is very likely to follow the way of a word once rich in meaning but now fast losing its particular significance, namely, democracy. We are fortunate, however, in educational administration to have a definition of our fundamental thesis which will ever guide us in our thinking and preserve the full significance of a postulate in the theory of state aid to schools. The definition is found in the report of The Educational Finance Inquiry and is as follows:

> To carry into effect the principle of "equalization of educational opportunity and equalization of school support" as commonly understood, it would be necessary (1) to establish schools or make other arrangements sufficient to furnish the children in every locality within the state with equal educational opportunities up to some prescribed minimum; (2) to raise the funds necessary for this purpose by local or state taxation adjusted in such manner as to bear upon the people in all localities at the same rate in relation to their tax-paying ability; and (3) to provide adequately either for the supervision and control of all the schools, or for their direct administration, by a state department of education.[1]

A careful reading and acceptance of the above quotation reveals that a better fiscal program for a state involves the objective

[1] Strayer, G. D. and Haig, R. M. *The Financing of Education in the State of New York*, p. 174.

measures of the following factors: (1) minimum program, (2) ability to support education, (3) adequacy of any program. The problem whose solution is attempted in this monograph is directly related to the first of these, the minimum program. So we must pause for a short time to define clearly the minimum program and to understand thoroughly its full implications.

The minimum program is best defined by stating those criteria that determine the selection of elements of such a program. These criteria are stated as follows:

1. An educational offering found in all the communities in the state when the equalization program takes effect should be included in the minimum program.
2. When, because of conditions over which the local community has little or no control, supplementary undertakings are necessary in order to make it possible to carry on any activity, chosen under the first principle mentioned above, these undertakings should be included in the minimum program.
3. When additional offerings are required in order to supply educational returns commonly expected from the minimum program but which, because of conditions over which the local community has little or no control, may not be expected to materialize, these additional undertakings should be included in the minimum program.
4. If there is reason to believe that the inclusion of any element in a minimum program will have any other than a salutary effect upon the educational offering in any community or will bring about harm that is out of all proportion to the good involved in including it in the burden to be equalized, it should be omitted from the minimum program.[2]

The foregoing clear-cut definition of the minimum program enables us to define the educational need of a district as the sum total of all the factors that affect the cost of the minimum program. This definition is valid regardless of the size and character of the district. We thus determine the educational need of an urban district or rural community by finding a relative measure of the cost of the minimum program.

In the past years the states have been satisfied with rough measures of educational need, such as number of teachers, number of pupils, expenditures of a community for education, and the like. These measures were considered adequate because the states were rather generally distributing state aid under the two principles of (1) rewarding for effort, and (2) satisfying an educational

[2] Mort, Paul R. *State Support for Public Schools*, p. 8.

need. The acceptance of the principle of equalization of educational opportunity soon revealed that these principles are mutually exclusive and that a state program of equalization must be based on the educational need. To the extent to which we reward for effort, just to that extent do we upset any system of equalization.

Since any equalization program demands a scientific measure of the educational need of the school districts, progress in this field could only be made as fast as adequate measures were developed. In a pioneer study by Mort,[3] we find a unique and scientific development of just such a measure. He accepts the pupil and teacher measures, and introduces certain corrections to make his two measures more accurate and in proper proportion to the cost of a minimum program. His measure is expressed in terms of the weighted pupil or the typical teacher, two measures, one of which is a multiple of the other. Most of his standards are based on the relation between teachers employed and average daily attendance. By use of the regression equation he is able to state the typical number of teachers for a district of given size; he thus determines a relative measure of the cost of the educational program, for educational costs are proportional to teachers' salaries. These standards may be developed for each type of school organization and, when properly weighted, summed to make a single measure. By this method a community's share in an equalization program can be stated as the sum of typical teachers for both high and elementary schools plus certain other costs known as Group 2 costs.

The measure of educational need briefly outlined above was a noteworthy contribution to the solution of the problem of distributing state aid. At once it opened the field for further researches and there are now three ways by which this measure may be supplemented: (1) equalizing the burden of capital outlay, (2) correcting the present measure of educational need in terms of cost of living, (3) developing an index of the cost of transportation.

In the Group 2 costs mentioned above, Dr. Mort included only the transportation costs. He made no attempt to arrive at a fundamental measure of the educational need represented by transportation costs. The actual expenditures were taken which gave a measure in terms of actual cost to be transmuted into relative cost measures such as typical teachers. This has the

[3] Mort, Paul R. *The Measurement of Educational Need.*

serious shortcoming of varying with the cost of the minimum program.

The importance of the transportation index is clearly stated in the following quotation:

> There is need for development of an adequate index for measuring the cost of transportation of pupils. In some communities transportation of pupils is necessary in order that the state's minimum program may be offered. The costs of such transportation are legitimate responsibilities of the state as a whole. They should be considered as a part of the cost of the minimum program. Up to this time, however, no adequate index of transportation cost has been developed. States that are seriously attempting to assume the responsibility for a satisfactory minimum program are handicapped by the lack of such an index.[4]

The importance of transportation of pupils in modern administrative policies is shown by a study on school attendance made by Reavis:

> Distance from school is the strongest factor influencing the attendance of pupils enrolled in the rural schools of the five Maryland counties included in this study. The data for this study did not furnish sufficient cases to make an exhaustive study of transportation. The study is concerned with transportation only as it is necessary to translate days transported into equivalent distances walked. But there seems to be very good evidence that transportation practically eliminates the distance factor.
>
> . . . . . .
>
> If the distance factor is strong enough to reduce the days attended one half at a distance of two miles from the school, it would seem that children living remote from school, without some means of transportation, do not have equal educational opportunities with those residing nearer. States interested in equal educational opportunity for country children have another opportunity here. Not only should state aid be used to supplement the general resources of weak rural districts, but something more should be done for that large group of children in all country schools who live at some distance from the school. A state cannot place a school within a quarter of a mile of every house; but it can encourage free transportation by removing legal restrictions and supplying liberal grants of state aid for transportation.[5]

[4] Mort, Paul R. *State Support for Public Schools*, p. 99.

[5] Reavis, George H. *Factors Controlling Attendance in Rural Schools*, p. 21.

In the State of New Jersey the state aid to counties is distributed to the districts on the basis of the legal quotas which comprise apportionments for teachers, transportation, and tuition; and the remainder of state aid, if there is any, is then apportioned to the districts on the basis of days' attendance. Those counties which do not have sufficient money to meet legal quotas receive the deficit from the state and sufficiently more to apportion three cents for every day's attendance in the county. In this manner every county is granted sufficient money from the state funds to provide at least for legal quotas and three cents for every day's attendance. This is the minimum; some counties receive far more, enough to pay from ten to more than fifteen cents per day's attendance.

The deficit mentioned above, which faces many counties when the legal quotas are made up, is to a large extent directly related to the costs of transportation of pupils. In the Education Bulletin for May, 1926, issued by the New Jersey State Department of Public Instruction, there is published a table which shows "Percentage of Total Day School Expenditures Devoted to the Major Current Items," for the school year ending June 30, 1925. The bulletin states in part:

> Many of the rural districts must provide transportation and pay tuition which cause their auxiliary agencies . . . to be higher in proportion to total expenditures than cities and urban districts.

An examination of the table mentioned above shows that median auxiliary agency expenditures are 16.5 per cent, 2.9 per cent, and .7 per cent, respectively, for the rural, urban, and city districts. Auxiliary agencies here included all costs of libraries, playground activities and equipment, lectures, net cost of school lunches, and transportation. It is unfortunate that the transportation costs are not here separated, but we have the report [6] from Ocean County for the year 1925–26 which shows that 10.8 cents of every school dollar was spent for transportation. For that same year we find that Ocean County lacked sufficient funds to meet legal quotas and was forced to draw on the state reserve fund, from which it received sufficient money to meet the deficit and pay 3.1 cents per day's attendance. During the same year, Atlantic County was able to meet its legal quotas and pay 17.5 cents per day's attendance.[7]

[6] Bulletin No. 1, Oct. 15, 1926, Ocean County Public Schools.

It is quite apparent from the above data with respect to New Jersey that the burden of transportation is very heavy in some counties in New Jersey. There is, indeed, a very great inequality in education portrayed here, for we must realize that every dollar expended in transportation means one dollar less for those activities, such as instruction, that are the vital part of education. Heavy transportation charges mean a program of education proportionately lessened in richness. To put it another way, there is no educational return in merely transporting pupils in a bus. It is simply a supplementary activity. If we refer to our criteria for selecting the elements that enter into a minimum program to be equalized by the state, we see at once that transportation comes under item two and is a part of the minimum equalization program. The state should, therefore, assume full responsibility for equalizing the burden of the costs of transportation.

The purpose of this study may now be expressed: A fiscal program for a state, based on the principle of equalization of educational opportunity, involves the transportation of school children to school at public expense. It is the purpose of this study to discover an index which would enable a state to apportion school moneys for transportation in a manner consistent with the above principle. Since this index is to supplement a fundamental and pioneer study in the measurement of educational need, it is quite necessary to discuss this measurement of need and its full significance.

SUMMARY

1. Equalization of educational opportunity involves (*a*) minimum program, (*b*) equality of burden, (*c*) adequate supervision and control of schools.

2. There are needed objective measures of (*a*) the minimum program, (*b*) ability to support education, (*c*) adequacy of any program.

3. Selection of the elements of a minimum program is based on (*a*) universality, (*b*) needed supplementary offerings, (*c*) supplementary educational offerings to reach a minimum program, (*d*) ultimate good.

4. Educational need is the sum total of all factors affecting cost of a minimum program.

[7] Education Bulletin, June, 1925, New Jersey Department of Public Instruction, p. 439.

5. The pioneer study by Mort discovers an adequate measure of educational need but neglects the three factors of (*a*) cost of living, (*b*) capital outlay, (*c*) transportation.

6. The study by Reavis shows the importance of transportation in removing the distance factor in attendance at rural schools in Maryland.

7. The heavy burden of transportation on rural districts in New Jersey is shown.

8. The problem of the present writer is defined as the derivation of a transportation index to supplement the previous study in measurement of educational need by Mort.

CHAPTER II

SELECTION OF CRITERION OR INDEPENDENT VARIABLE

Since it is the purpose of this study to discover a measure of transportation need that will assist in the distribution of state aid on a basis of equalization, it becomes necessary to secure some recent and reliable data from which to derive our index. For this reason those data were chosen that were immediately available to the writer in the form of annual reports and which did not at once require the issuance of a questionnaire. The New Jersey School Reports were available for a period of years and contained some very important data that could be secured at once. These school reports supplemented by census reports for 1920 are the sources of data from which the standards were derived.

It may be well to point out at this time that the writer was endeavoring to secure a general solution to the problem of equalization of the burden of transportation which could be applied wherever there was a need for such an index. It is obviously quite impossible to isolate such an index from the general problem of equalization and, indeed, in the application of the index it must be an integral part of the whole program of equalization of educational opportunity. The index which will be derived here is not proposed as a solution for the transportation problem in New Jersey. It is a solution only as it becomes a part of the bigger problem, equalization of educational opportunity in the State of New Jersey. The index can not be divorced from the fundamental task of the measurement of educational need in the state.

In view of the fact that the solution to the general problem is to be based on material from the State of New Jersey, it becomes necessary to give here the "Rules for Transportation" effective in this state. These rules will be found in a letter to the county superintendents dated June 8, 1916, which reads as follows:

To County Superintendents:

The State Board of Education has taken no action in regard

to the first four rules concerning transportation, which received the approval of the County Superintendents:

"In construing the necessity for, method and cost of, transportation for pupils who may live remote from public school facilities and accommodations, the following rules shall govern under ordinary and usual conditions.

"1. All transportation arrangements that contemplate the apportionment from State funds of 75 per cent of the cost must have the written approval of the County Superintendent of Schools.

"2. When high school pupils live within two and one-half miles of high school facilities and accommodations, which facilities and accommodations are interpreted to mean the site of the approved high school, transportation shall not be deemed necessary. In establishing a transportation route it should be routed to accommodate the largest number of high school pupils entitled to approved transportation, and no high school pupil should be required to walk more than two miles to the nearest accessible point on said route.

"3. When elementary school pupils live within two miles of elementary school facilities and accommodations, which facilities and accommodations are interpreted to mean the site of the elementary school, transportation shall not be deemed necessary. In establishing a transportation route it should be routed to accommodate the largest number of elementary school pupils entitled to approved transportation, and no elementary school pupils should be required to walk more than one and one-half miles to the nearest accessible point on said route.

"4. Transportation within the distances prescribed in rules two and three, and transportation without the approval of the County Superintendent may be provided by a board of education. In all such cases, however, the County Superintendent shall make no apportionment of State Moneys for 75 per cent of the cost of said transportation."

As to the above rules, you may use them as tentative or suggestive in your approval of transportation contracts. Whenever conditions seem to you to be such that you should modify these rules to meet a local condition, you should use your best judgment in regard to such modification.

I am very desirous that transportation expenses should not increase any faster than is absolutely necessary.

The State Board of Education passed the following rules on Saturday, June 3rd, 1916:

"Rule 1. It shall be the duty of the local board of education to report to the County Superintendent on or before August fifteenth in each year all such high school pupils and such other pupils of elementary grades as may be entitled to transportation; such report shall give the age and grade of each pupil, the name of parent or guardian; shall designate the school to be attended

and the distance and the method of proposed transportation; and said report shall be accompanied by a sketch of proposed route with the residence of each pupil to be provided for indicated thereon.

"Rule 2. In establishing or readjusting transportation routes, it is recommended that boards of education advertise for competitive bids.

"Rule 3. All contracts for transportation routes and agreements for individual transportation shall be filed with the County Superintendent of Schools for approval on or before September first in each year and shall be accompanied by a certified copy of the minutes of the board of education authorizing same. All such contracts shall be accompanied by a suitable bond for at least the full amount of the contract signed by at least two responsible sureties."

Of course it is understood, without action of the State Board of Education, that all transportation arrangements that contemplate the apportionment from State funds of 75 per cent of the cost must have the written approval of the County Superintendent of Schools.

Very truly yours,
C. N. KENDALL

The New Jersey Annual Report for 1923, the most recent at the initiation of this study, and the U. S. Census Report for 1920 yield material which is presented in Table I. It is here that the search for the index begins and from these data it is necessary to select the criterion[1] which is stated as the problem of this chapter.

The derivation of a measure of transportation resolves itself into the selection and definition of two variables. One of these variables may be called the dependent variable, or Y value, and it will be associated with the need for transportation. The other variable which will be called the independent variable, or X value, may be associated with those causes that bring about a need for transportation. It should have a high correlation with the need of transporting children to school. A glance at the study by Mort would show that his Y value may be thought of as the number of teachers, and his X value as the average daily attendance. The number of teachers is proportional to the cost of education and hence to the educational need—it is the Y value or dependent variable; the average daily attendance has a high correlation with the number of teachers—it is the X value or independent variable.

Returning to the definite problem of measuring the transporta-

[1] Criterion may be defined here as the standard by which a correct judgment can be made with respect to the need for transportation.

tion need for the counties of New Jersey, it is assumed for the time being that the transportation need or cost of transportation is proportional to the number of children actually transported to school in accordance with the Transportation Rules quoted above. Since the counties vary greatly in the size of school population, we place them on a comparable basis by determining what percentage of the average daily attendance is transported to school. This is the Y value or dependent variable. Needless to say, this measure will be refined later. Knowing the percentage of pupils transported to school and taking this as the Y value or transportation need of the counties, it becomes necessary to select the X value or independent variable that has a high correlation with the transportation need. If it is possible to find such a variable, it may be used as a criterion by which to determine the ratio or percentage of pupils transported for a given county; this in turn would yield the number of pupils transported, the transportation need.

The first measure or criterion that suggests itself is the density of total population, which can be secured from the U. S. Census returns of the counties of New Jersey. This measure, with that of the transportation need mentioned in the above paragraph, gives the distributions found in Table I.

The two variables whose relation is to be examined are (1) the percentage of pupils transported to school, the Y value and measure of transportation need; (2) the density of total population, the X value, the criterion which is assumed to be related to transportation need. In order to discover the relation quickly, the correlation was determined by means of Spearman's Rank Order formula. It is $-.87$ and a truly significant correlation which justifies to some extent the previous assumptions relative to density and transportation. A more refined measure is then taken of the correlation, Pearson's Product-Moment Method or straight-line relationship, and the correlation drops to $-.42$.

At first thought this is startling and somewhat disappointing, but Otis has this to say of Spearman's formula used above, when the rank formula gave a higher value of r than the Pearson r

> The two rank methods give results agreeing fairly closely with one another, but both are likely to differ appreciably from the true value of r by the standard method. For this reason the rank methods are not recommended except in case of non-linear relationship; i.e., those in which the line of relation is

TABLE I

RELATION BETWEEN FUNCTIONS OF DENSITY AND PERCENTAGE OF A.D.A. TRANSPORTED, 1923

County	A.D.A.[1]	Number Pupils Transported [1]	% A.D.A. Transported	Density of Population [2]	$\sqrt[2]{\text{Density}}$	$\sqrt[3]{\text{Density}}$	$\sqrt[4]{\text{Density}}$	log Total Density	X $\log\left[\frac{\text{A.D.A.}}{\text{Area}}\right]$	Y $\frac{\text{P.T.}}{\text{A.D.A.}}\sqrt{\frac{\text{Area}}{\text{Bldgs.}}}$	$\sqrt{\frac{\text{Area}}{\text{Number Bldgs.}}}$
Atlantic	16,249	1,311	8.608	147.5	12.12	5.283	3.49	2.17	1.46	21.50	2.50
Bergen	45,693	3,356	7.344	889.0	29.81	9.615	5.46	2.95	2.29	8.65	1.77
Burlington	13,332	4,292	32.190	100.3	10.01	4.646	3.17	2.00	1.21	93.90	2.89
Camden	29,853	1,438	4.816	858.1	29.29	9.430	5.41	2.93	2.13	5.75	1.19
Cape May	4,429	590	13.319	73.4	8.56	4.187	2.93	1.87	1.22	29.50	2.21
Cumberland	11,409	2,047	17.941	122.7	11.07	4.969	3.33	2.09	1.36	43.30	2.41
Essex	110,443	2,106	1.906	5134.6	71.65	17.252	8.47	3.71	2.94	1.55	.82
Gloucester	9,559	1,952	20.420	145.3	12.05	5.257	3.47	2.16	1.46	40.10	1.96
Hudson	91,159	1,371	1.503	14631.5	120.93	24.458	10.79	4.17	3.33	.92	.61
Hunterdon	5,101	630	12.330	75.3	8.67	4.227	2.95	1.88	1.07	28.00	2.27
Mercer	22,645	1,942	8.575	707.4	26.59	8.903	5.16	2.85	2.00	13.50	1.58
Middlesex	28,888	2,266	7.844	520.3	22.81	8.042	4.78	2.72	1.97	12.80	1.63
Monmouth	20,930	3,183	15.207	219.1	14.79	6.028	3.85	2.34	1.64	27.80	1.83
Morris	14,305	1,957	13.680	174.1	13.19	5.583	3.63	2.24	1.48	26.40	1.93
Ocean	4,467	1,138	25.473	34.8	5.89	3.264	2.43	1.54	.85	83.00	3.26
Passaic	43,536	1,639	3.762	1322.3	36.36	10.975	6.03	3.12	2.35	5.27	1.40
Salem	5,865	1,352	23.052	106.6	10.32	4.741	3.21	2.03	1.23	53.00	2.30
Somerset	8,336	882	10.580	157.3	12.54	5.398	3.54	2.20	1.44	20.90	1.98
Sussex	4,299	943	21.935	47.1	6.86	3.611	2.62	1.67	.91	54.70	2.49
Union	36,119	1,327	3.682	1943.3	44.08	12.479	6.64	3.29	2.54	3.68	1.00
Warren	7,667	822	10.707	124.5	11.15	4.955	3.37	2.10	1.33	21.20	1.98
Total	534,294	36,544									

[1] New Jersey School Report, 1923.

[2] U. S. Census, 1920.

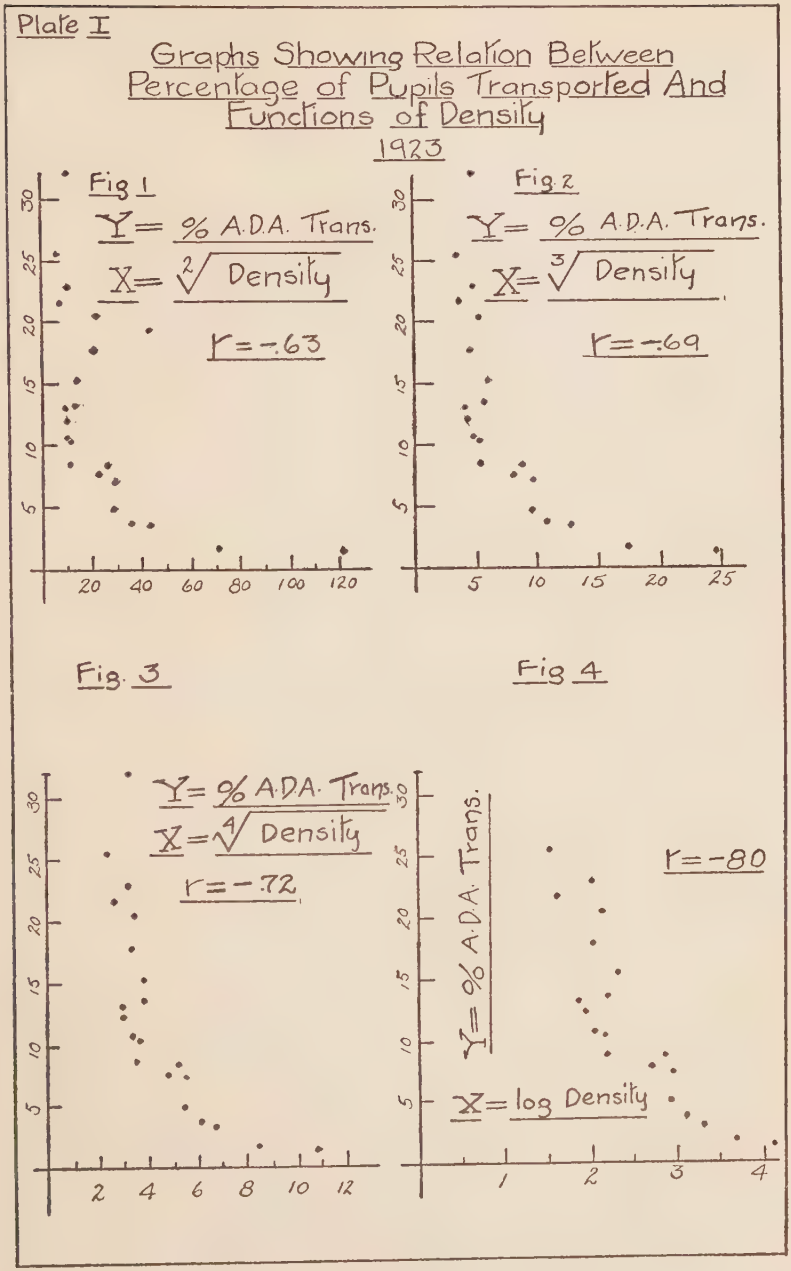
Plate I
Graphs Showing Relation Between Percentage of Pupils Transported And Functions of Density
1923
Fig 1
Y= % A.D.A. Trans.
X= ²√ Density
r=-.63
Fig 2
Y= % A.D.A. Trans.
X= ³√ Density
r=-.69
Fig 3
Y= % A.D.A. Trans.
X= ⁴√ Density
r=-.72
Fig 4
Y= % A.D.A. Trans.
X= log Density
r=-.80

PLATE I

curved. In these cases the rank methods are presumed to show what the correlation would be if the line of relation were straight.[2]

The conclusion reached from a careful reading of this quotation is that the relation between density of population and transportation need is a non-linear one whose true value is not shown by the Pearson *r*. The rank order correlation of −.87 is presumed to be the correlation that would exist if the line of relationship were straight. It is thus apparent that density is related to transportation and the next step is to discover this relationship. Extracting the square root of the density and again correlating against the transportation need, a correlation of −.63 by the Pearson method is found. This distribution is shown in Table I. A correlation of −.63 appears to be quite promising and the process is carried still further by correlating in turn the cube root and fourth root of the density against the transportation need; these applications yield respectively Pearson *r*'s of −.69 and −.72. Finally the log of the density of population is correlated against the transportation need and a Pearson *r* of −.80 is found. All these distributions are shown in Table I.

In Plate I is shown graphically the result that is obtained as one takes the various functions of the density for the independent variable or criterion. It appears that the relation between density and transportation is a curvilinear one, but a rectilinear relationship is approached as respective functions of the density are taken.

The result thus far secured may be stated as follows: there is a fairly high correlation between the log of the density of total population and the percentage of pupils transported to school in the counties of New Jersey. The function of the density may be taken as the criterion and the percentage of pupils transported as the transportation load. An examination of the criterion will reveal at once a way in which it should be improved. Since this is a problem of educational administration, it is evident that any educational need should be based on the density of school population rather than the total density.

At this point in the study it was once thought desirable to introduce some measure of the relation of school buildings to density in the index. Two arbitrary indices were proposed involving these relationships. The first relationship proposed would give an independent variable or criterion as shown on page 16.

[2] Otis, Arthur S., *Statistical Method in Educational Measurement*, p. 212.

Plate II

Graphs Showing Relation Between Percentage A.D.A. Transported And Indices 1923

Fig. 1

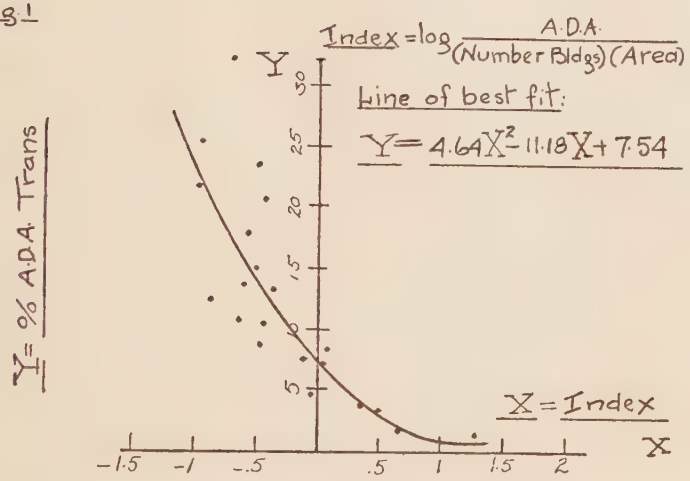

Fig. 2

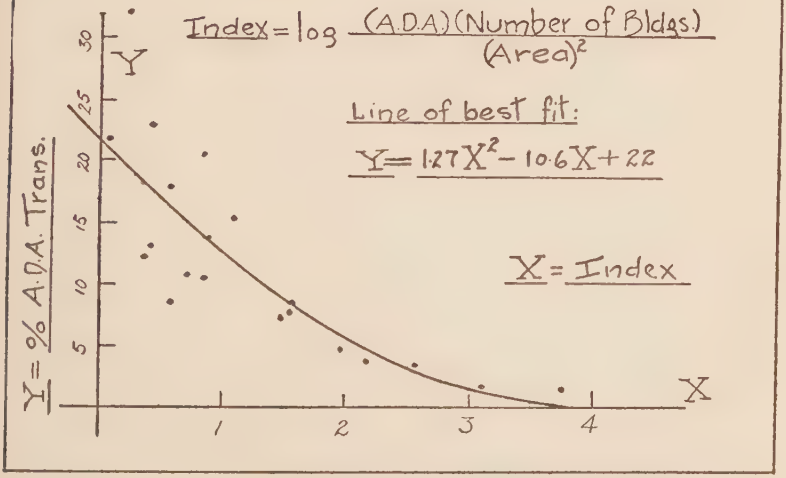

PLATE II

$$X = \log \left(\frac{\text{A.D.A.}}{\text{Number of School Buildings} \times \text{Area}} \right);$$

the second relationship proposed would give a criterion as follows:

$$X = \log \left(\frac{\text{A.D.A.} \times \text{Number School Buildings}}{(\text{Area})^2} \right).$$

Both of these indices were plotted against the transportation need and are shown in Plate II.

An examination of Plate II will show that the two indices have a strong statistical interest, for they appear to correlate highly with the transportation need. The equation of the line of best fit is given in each case but the writer reserves for a later part of this study a discussion of just how this line and its equation were determined. A careful analysis of the indices will, however, reveal a fallacy which would destroy the whole system of equalization if it were based on these indices. It is for this reason alone, to show adequately a pitfall in the search for a valid index, that the writer presents this plate.

Any program of equalization of educational opportunity must have for the criterion a variable which is not susceptible to local control or policies as far as the minimum program is concerned. The criterion in the case of Mort was the average daily attendance, a figure which represents that factor which causes the educational need, the employment of teachers. In the case under discussion, if the buildings were allowed to enter into the X value, then a county could affect its location on the X-axis by its policy with respect to number and type of school buildings, which in turn would react in the transportation need. The criterion, or X value, should be absolutely beyond the control or manipulation of any county by any local policy, and for this reason the two indices in Plate II are rejected, and the log of density of school population is retained as the independent measure of the causal factor in transportation.

The Y value, or measure of transportation need, has been defined as the percentage of pupils transported to school. In any problem involving transportation there are really two factors involved: (1) what is to be transported; and (2) how far it must be transported. The measure of transportation need may, therefore, be refined if one introduces some system of weighting that will be related to the distance children travel to school. It is here that

Plate III

Graph Showing Relation
Between
Density of School Population
And
Transportation
In New Jersey Counties
1923

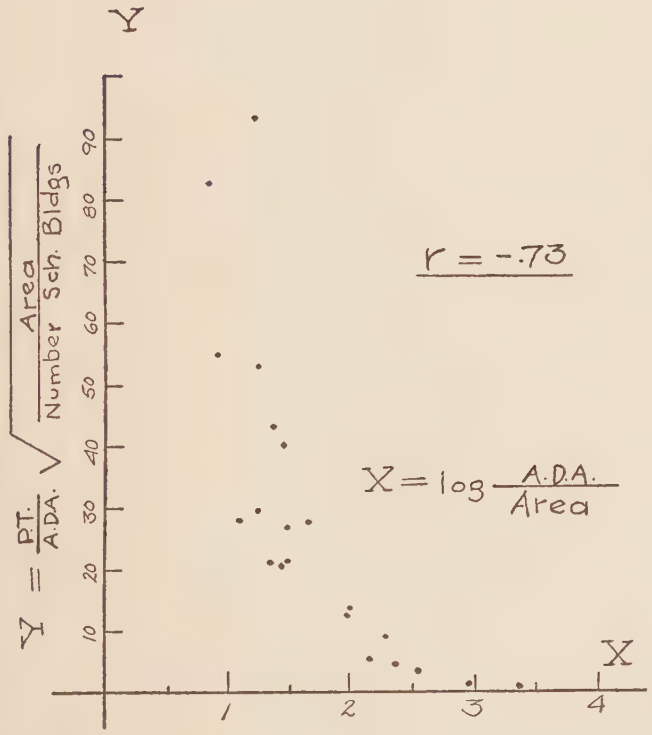

Plate III

one finds a legitimate use for the factor which was rejected from the X value, namely, buildings. If the square miles per building be determined and then the square root be extracted, there will result for each county a linear distance that is comparable to the distance that the children, on the average, travel to school in that county. This weighting also has the advantage of being related to the administration policy of each county with respect to number of school buildings.

The percentage of pupils transported to school may be accepted at once as one measure of the transportation load. The square root of the square miles per school building is a figure which inspection of Table I will show to be closely related to the distance children travel to school. For example, for the urban counties of Essex and Hudson this figure is found to be .82 miles and .61 miles, respectively; for the rural counties of Ocean and Burlington, this figure is found to be 3.26 miles and 2.89 miles, respectively. The product of the percentage of children transported to school and the square root of the square miles per building will yield a figure quite similar to a pupil-mile measure which may be taken as the measure of the transportation load being carried by the particular county. It remains to be seen how the measure will correlate with the log of the density of school population.

In Plate III there is shown a plot of these two variables. The product-moment correlation for these two variables is $-.73$. The reliability of this method may be checked by examining the situation that is found for a period of years in the counties of New Jersey with respect to these variables which are defined as follows:

$$Y = \frac{\text{Pupils Transported}}{\text{A.D.A.}} \sqrt{\frac{\text{Area of County}}{\text{Number of School Buildings}}}$$

$$X = \log\left(\frac{\text{A.D.A.}}{\text{Area}}\right)$$

The correlation for the years 1921, 1922, 1923, 1924, 1925 are as follows $-.72$; $-.74$; $-.73$; $-.79$; $-.78$; with an average correlation of $-.75$. It should be borne in mind that one has here only a measure of the linear relationship, and these correlations are to be taken only as an indication of the validity and reliability of the proposed criterion or independent variable, the log of the density of school population. An examination of Plate III will show the distribution to be decidedly curvilinear and the final solution

will demand a more refined technique than that of correlations and straight-line regression.

The conclusion to be reached in this chapter is that the log of the density of school population is a valid and reliable criterion by which the transportation need of a county may be predicted.

In the search for the factor in the transportation problem, it must be borne in mind that it is not desirable or possible to discover a factor that will yield a correlation of 1.00 with the present situation. To do so would be merely to duplicate the inequalities that now exist. If there is a fundamental law operating to produce a transportation need, it will be so obscured by local practice and policy that it can not clearly be set forth with all the precision of a law in a natural science. In the field of social science there can be only an approximation of the laws that are believed to be operating.

The log of the density of school population is chosen as a criterion for the following reasons:

1. It has a high correlation with the transportation load.
2. It is a valid and reliable measure over a period of years.
3. It is beyond the control of local policy.
4. The minimum transportation program may rightfully be based on it, for it withstands a common sense analysis.

SUMMARY

1. The sources of data are New Jersey School Reports and the U. S. Census Report for 1920.

2. The transportation index can not be divorced from the whole problem of equalization.

3. Statement of Rules for Transportation in New Jersey is necessary to the study.

4. It must be expected that the transportation law, if there is one, will be obscured by local policy, and no variables will or should correlate perfectly.

5. The log of the density of school population is chosen as the independent variable or criterion, and percentage of pupils transported multiplied by the square root of square miles per building is chosen as the measure of transportation need.

CHAPTER III

CURVE FITTING AND SELECTION OF A CURVE

The specific task for this chapter is to discover the law of relation between the two variables selected in Chapter II, namely, the transportation load and the log of the density of the school population. This law when expressed in mathematical symbols will make it possible to determine the transportation need of a given county when the criterion, the density of school population, is known. The transportation need in each case will be a figure similar to an average for all counties of a given density. It is analogous to the "typical teachers" figure used by Mort in his study of educational need. At this point in the study, the New Jersey School Report for 1924 became available and the investigations of this chapter are based on distributions for 1924 of the two variables chosen in Chapter II.

In mathematical language our purpose may be defined as the search for an empirical formula. Formulas may be grouped into two classes, rational and empirical. These two classes have no sharp dividing line. Rational formulas are those such as are met in physics and other natural sciences. They are deduced by a process of mathematical reasoning from the elementary laws of the science to which they belong. These formulas contain certain constants whose determination sometimes is a matter of great scientific importance. Empirical formulas are inferred from experiment and observation and are not deduced theoretically. They also have certain constants to be determined just as if they were rational formulas. These constants are called the empirical constants.

Empirical formulas arise when an experiment reveals that some law is operating which can not be deduced theoretically but can be expressed simply in mathematical symbols. The complexity of the situation may be such that the investigator can not trace the connection between cause and effect and is forced to search blindly for a functional relation between them. It some-

TABLE II

DERIVATION OF VARIABLES X AND Y FOR SCHOOL YEAR 1924

County	A.D.A.[1]	Pupils Transported[1]	% A.D.A. Transported	Area[2]	$\frac{\text{A.D.A.}}{\text{Area}}$	Number[1] Bldgs.	$\sqrt{\frac{\text{Area}}{\text{Bldgs.}}}$	Y = % A.D.A. Trans. $\times \sqrt{\frac{\text{Area}}{\text{Number Bldgs.}}}$	X = $\frac{\text{Log A.D.A.}}{\text{Area}}$
Atlantic	16,771	1,580	9.421	569	29.47	89	2.528	23.82	1.46945
Bergen	49,097	3,792	7.723	237	207.16	178	1.153	8.89	2.31631
Burlington	14,086	3,980	28.255	815	17.28	98	2.883	81.48	1.23763
Camden	31,384	1,638	5.219	222	141.37	155	1.196	6.24	2.15036
Cape May	4,654	724	15.556	265	17.56	54	2.215	34.46	1.24458
Cumberland	11,597	2,283	19.686	500	23.19	88	2.383	46.92	1.36538
Essex	113,745	1,897	1.667	127	895.62	196	.804	1.34	2.95212
Gloucester	10,499	2,066	19.678	332	31.62	88	1.942	38.22	1.50001
Hudson	92,444	1,151	1.245	43	2149.86	117	.606	.75	3.33241
Hunterdon	5,123	604	11.790	437	11.72	87	2.241	26.42	1.06904
Mercer	23,734	2,146	9.041	226	105.01	88	1.602	14.49	2.02126
Middlesex	30,621	2,628	8.582	312	98.14	126	1.573	13.50	1.99187
Monmouth	21,501	3,331	15.492	479	44.88	145	1.817	28.15	1.65212
Morris	14,825	2,280	15.379	475	31.21	126	1.941	29.86	1.49430
Ocean	4,787	1,241	25.924	637	7.51	59	3.285	85.18	0.87592
Passaic	45,142	1,517	3.360	196	230.31	98	1.414	4.75	2.36232
Salem	5,984	1,550	25.902	343	17.44	65	2.297	59.50	1.24170
Somerset	8,677	1,020	11.761	305	28.43	77	1.990	23.40	1.45382
Sussex	4,508	889	19.720	529	8.52	75	2.655	52.37	0.93052
Union	37,754	1,483	3.928	103	366.53	111	.963	3.60	2.56412
Warren	8,186	909	11.104	362	22.61	90	2.005	22.26	1.35436
Total	555,119	38,709							

[1] New Jersey Annual Report, 1924.

[2] U. S. Census, 1920.

times happens that such a relation may lead to a theory on the nature of the connection which may be tested by carefully controlled experiments.

The search for an empirical formula always involves the selection of a mathematical expression. There is no general rule to guide here and often the problem demands considerable ingenuity and a good "hunch" to arrive at a simple and satisfactory solution. It is always helpful to plot the data first, and then a careful examination may suggest the type of expression to select, such as straight line, parabola, and the like. The type of expression once selected, the calculation of the empirical constants may proceed directly.

Two fundamental methods can be employed here: (1) the method of least squares, (2) the method of moments. The first method is employed in this chapter, and it is also the method used in the derivation of the regression line so familiar to all in the study of correlations. The regression line is sometimes called the line of best fit. This means that the regression line for a given scatter diagram is such that the sum of the squares of the deviation from it is a minimum. The reader should note that the line of best fit refers to the fitting of straight lines only, in the above illustration. For every type of line there is a line of best fit, and the scatter diagram mentioned above might better be fitted by a curve such as the parabola. Only careful study will determine the solution.

In Plate IV a distribution of the variables under investigation is shown for the school year 1924.

The first formula that suggests itself is that of a straight line. It is an equation of the first degree, $y = mx + c$ being the general form. The empirical constants to be determined here are m and c. The coefficient of correlation for the distribution is $-.789$ and the regression line is

$$Y = -29.4X + 80$$

The empirical constants are -29.4 and 80. The regression line, or straight line of best fit, has been drawn, and it can be seen at once that it is not a satisfactory solution to our problem. In fact, it is quite apparent that the distribution here is a curvilinear one and a curved line must be chosen for the next attempt.

The first curve that was chosen in an attempt to select the

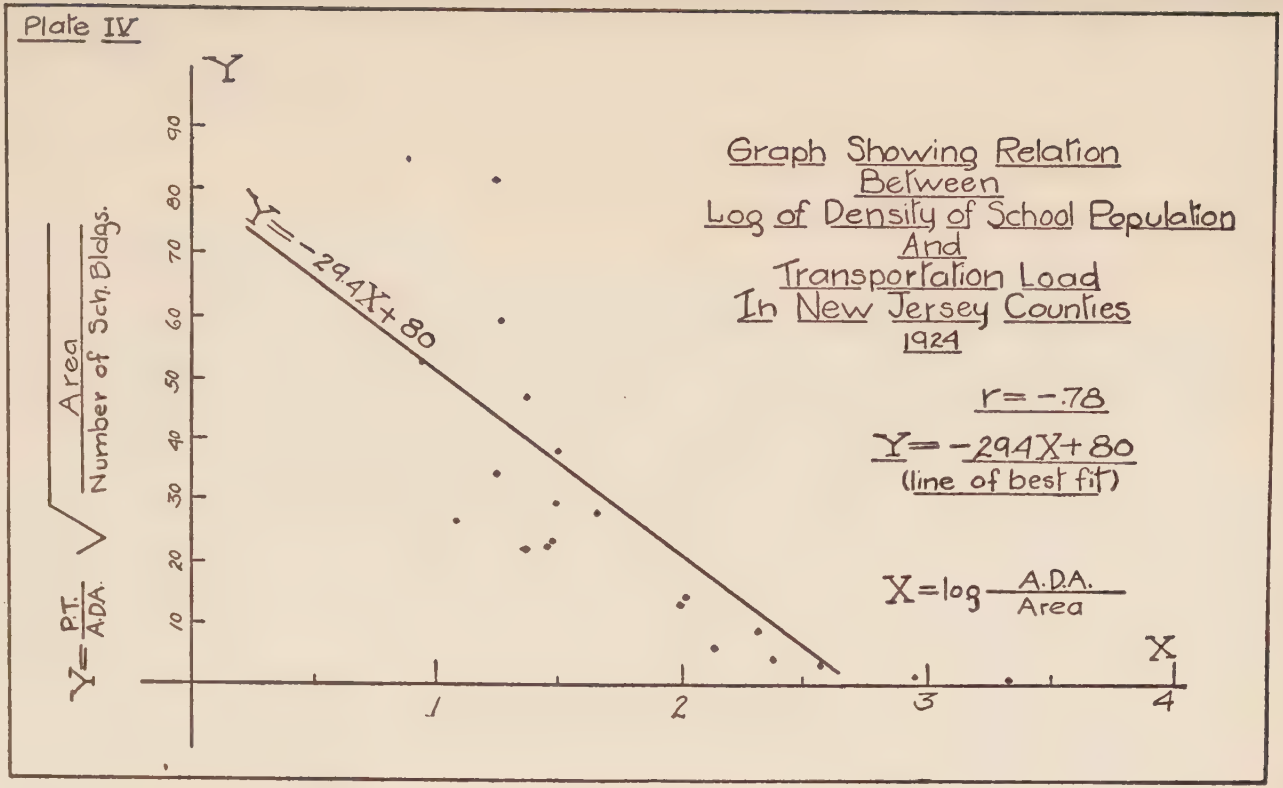
Plate IV
Y
90
80
70
60
50
40
30
20
10
Y=-.294X+80
Graph Showing Relation
Between
Log of Density of School Population
And
Transportation Load
In New Jersey Counties
1924
r=-.78
Y=-.294X+80
(line of best fit)
X=log A.D.A. / Area
Y=P.T./A.D.A. √ Area / Number of Sch. Bldgs.
X
1
2
3
4

PLATE IV

proper mathematical expression for our empirical formula was the parabola whose general equation is

$$Y = ax^2 + bx + c,$$

and the empirical constants to be calculated are a, b, and c. The calculation of these constants is a rather difficult process, and knowledge of the process [1] is not necessary for the understanding of the application of the curve to the problem of this chapter. The constants are determined to be as follows:

$$a = \quad 27.1$$
$$b = -138.36$$
$$c = \quad 175.99$$

and the desired equation may be written

$$Y = 27.1X^2 - 138.36X + 175.99$$

In Plate V is shown the distribution for 1924 and the curve whose equation was just determined has been drawn on it.

Examination will show that the parabola appears to be a good fit in the range from $X = 1$ to $X = 2$, very poor from $X = 2$ to $X = 3$, and bad from $X = 3$ upward. The curve cuts the X-axis in two points which may be determined by setting $Y = 0$ and solving the quadratic; these points are $X = 2.4$ and $X = 2.72$. Between these points the measure of transportation need is a negative quantity which, needless to say, is undesirable. The minimum value for the transportation need may readily be determined by calculus: extracting the first derivative and then setting it equal to zero and solving it gives a minimum at $X = 2.55$. This value when substituted in the parabolic equation gives $Y = -.61$. The whole situation may be stated as follows: the transportation need would decrease, in passing from places of small density of school population to those of greater, until it reached zero at $X = 2.4$, from 2.4 to 2.72 it would be negative in value, and from 2.72 it would increase rapidly. It is therefore impossible to accept this mathematical expression for a solution to our search for an empirical formula.

The next mathematical expression to be selected was a general one for power functions,

$$y = ax^n$$

[1] The reader who is interested in this process may consult Reitz, H. L., *Handbook of Mathematical Statistics*, p. 65.

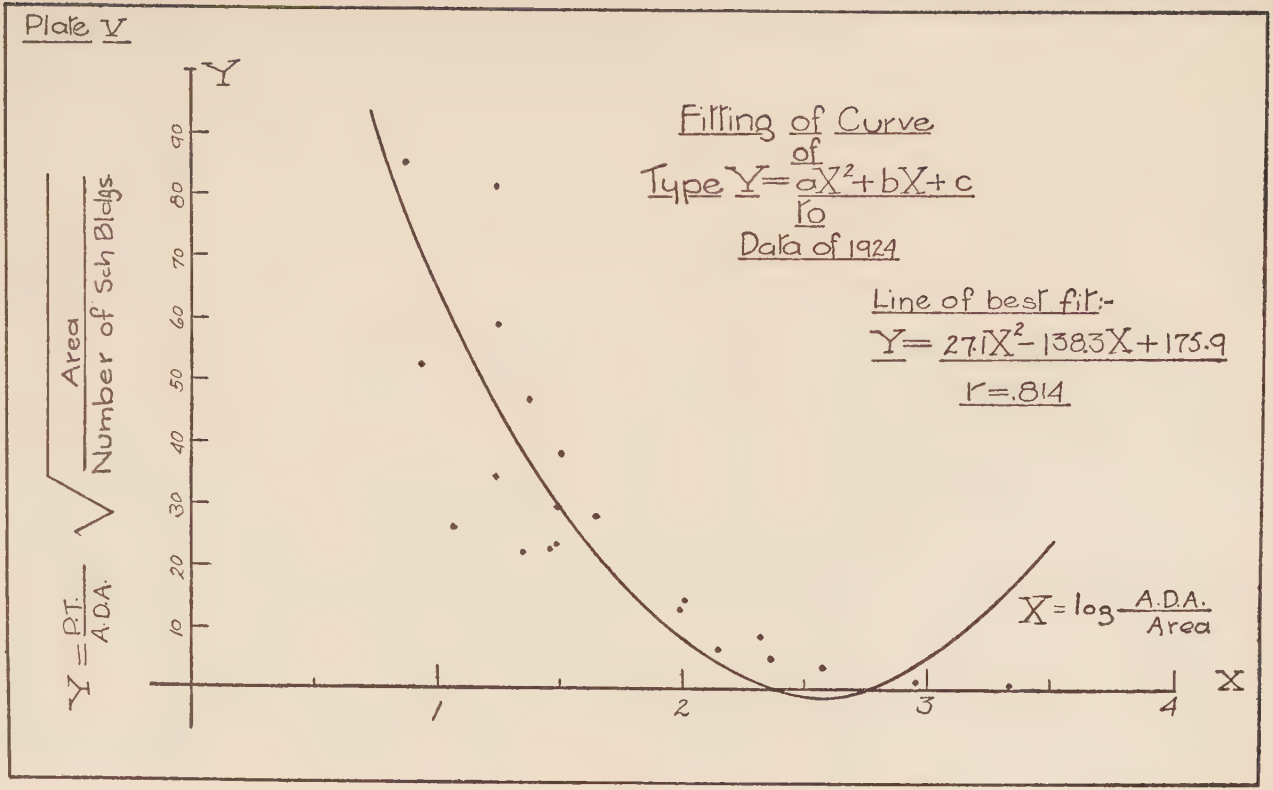
Plate V
Y
Fitting of Curve
of
Type Y = aX² + bX + c
to
Data of 1924
Line of best fit:-
Y = 27.1X² − 138.3X + 175.9
r = .814
Y = P.T. / A.D.A. √ Area / Number of Sch Bldgs
X = log A.D.A. / Area
X
10
20
30
40
50
60
70
80
90
1
2
3
4

PLATE V

where the empirical constants are a and n. If the log is taken of both sides of this equation, the result will be as follows:

$$\log Y = \log a + n \log X$$

An examination of this equation shows that it is the equation of a straight line and suggests a ready means of testing our assumption that this type of curve will be the desired empirical formula. If it is the desired one, then a plot of the logs of Y and X should be a straight line. The logs were plotted and approximated a straight line (with a correlation of $-.92$) whose regression equation is

$$y = 1.908 - 3.211x$$

Reversing the process of taking logs on this equation gives the general equation desired,

$$Y = 80.91X^{-3.21}$$

In Plate VI there is shown again the distribution of variables for 1924 on which the curve, whose equation has just been determined, is drawn.

It is now necessary to examine this curve, to see how well it fits the data. At first glance it appears to fit very well and is, in some respects, a much better fit in certain regions of the curve than the former curve, the parabola. This is more apparent in the region about $X = 2$ and upward. The reader will recall that in this region the parabola was a very poor fit for practical purposes, for it crossed the X-axis twice and rose sharply after the point $X = 2.72$ had been reached. In the lower region of the curve, below 2, the two curves on casual glance appear to fit equally well, but this is not true. The formula for the curve under discussion may be written

$$Y = 80.91 \frac{1}{X^{3.21}}$$

and analysis of this will reveal a fundamental weakness. As X approaches zero in value, the value of Y will approach infinity, and hence the curve in the region below $X = 2$ begins to rise very rapidly, making the deviations quite large as the X value becomes smaller. As X approaches zero in the parabola, Y approaches 175.99 in value. It is, therefore, safe to say that the fit for the curve under discussion is poor in the region below $X = 2$, and the curve has a weakness here that makes it undesirable for use in our

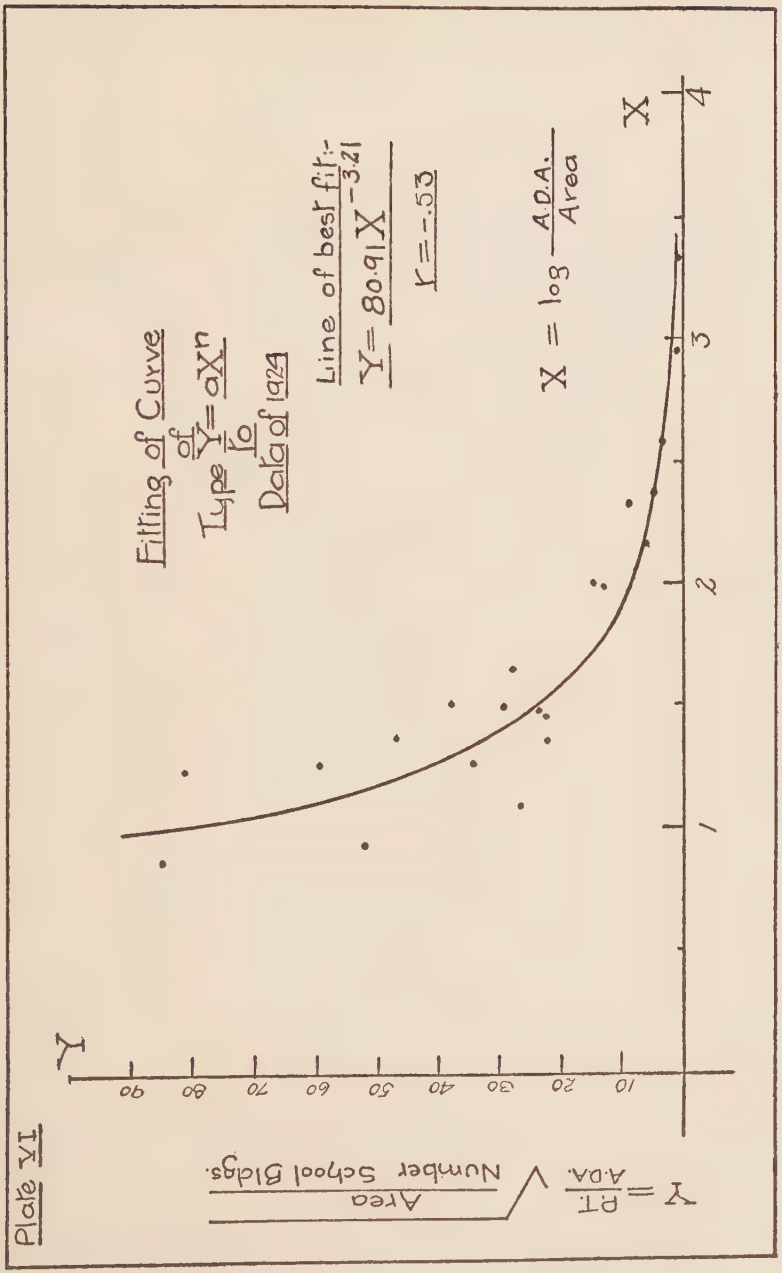
Plate VI
Fitting of Curve of Type Y = aX^n To Data of 1923
Line of best fit:-
Y = 80.91 X^-3.21
r = -.53
X = log A.D.A. / Area
Y = RT / A.D.A. √ Area / Number School Bldgs.
Y
X
10
20
30
40
50
60
70
80
90
1
2
3
4

PLATE VI

problem. A combination of the good features of the two curves discussed thus far should give a good solution.

The next mathematical expression which may be assumed as the solution to the problem under discussion is

$$Y = be^{ax}$$

The same method can be used here as was used in deriving the previous curve for simplifying the procedure, and a similar test can be applied to test the validity of the assumption. The method is as follows, and empirical constants to be determined are b and a:

$$\begin{aligned} Y &= be^{aX} \\ \log Y &= \log b + a\ (\log e)\ X \\ &= \log b + .434\ aX \end{aligned}$$

Again the desired equation is that of a straight line, and if this curve is a good fit, the log of Y, the transportation load, plotted against X, the log of the density of school population, will result in approximately a straight line. If such be the case, the straight line itself may be determined by correlations and regressions, an elementary method. In Plate VII the log of Y, the transportation need, has been plotted against X, the log of density of school population for the same school year of 1924.

A glance is sufficient to show that the distribution is a linear one; in fact, the correlation here is $-.96$, one that is remarkably high and bearing promise. Using the regression method a straight line of best fit to these data is secured. Its equation is

$$y = -.799X + 2.62,$$

and reversing the operation of taking logarithms, the desired equation may be written

$$Y = 416.87\, e^{-1.84X}$$

or in the straight line form,

$$\log Y = 2.62 - .799\ X$$

In Plate VIII are shown the data for the year 1924 plotted again, and the curve whose equation was just determined has been drawn.

A glance will reveal that the curve appears to combine the good features of the two previous curves and none of the bad features. It is a good fit throughout its entire length, crossing the Y-axis

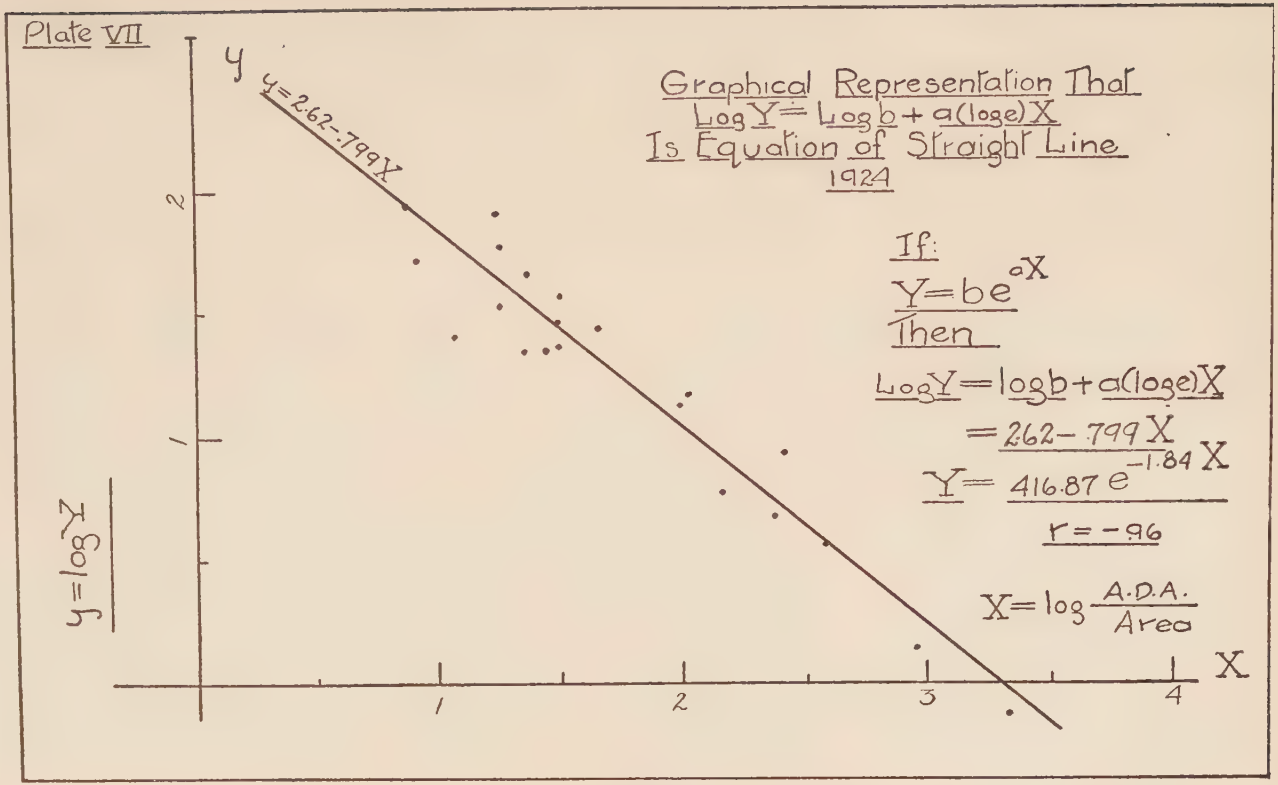
Plate VII
Graphical Representation That
Log Y = Log b + a(log e) X
Is Equation of Straight Line
1924
y = 2.62 - .799 X
If:
Y = be^aX
Then
Log Y = log b + a(log e) X
= 2.62 - .799 X
Y = 416.87 e^-1.84 X
r = -.96
X = log A.D.A. / Area
y = log Y
y
X
1
2
3
4

PLATE VII

at the point $x = 416.87$, and being asymptotic with respect to the x-axis. In other words, it does not have the bad feature of the second curve in the region below $x = 2$, or the bad characteristics of the parabola above the value $x = 2$ where it crossed the x-axis twice. It does seem to be a happy combination of the first part of the parabola and the final part of the second curve.

It is possible to derive a measure which will give a measure of the goodness of fit of the three curves which in turn have been assumed as solutions to the problem of this chapter. This measure is known as the generalized coefficient of correlation whose formula may be expressed as follows:[2]

$$r = \sqrt{1 - \frac{\Sigma(Y - \check{Y})^2}{N\sigma_Y^2}}$$

where Y equals the measure of transportation load for a given county and $\check{Y}$ is the equation value or predicted value. Using this formula and calculating the coefficient of correlations for the three lines they are found to be as follows:

(1) For the parabola $r = .814$
(2) For the function $Y = ax^n$ $r = .530$
(3) For the function $y = be^{ax}$ $r = .816$

It is now safe to say that the third curve is a solution to the problem of this chapter, which was the selection of an empirical formula. It is selected, for a careful examination of Plate VIII will show that this curve is an excellent fit to these data and the conclusion is verified by the correlation of .816, which is high indeed for a problem of this nature. This correlation may be interpreted to mean that the prediction based on this curve correlates .81 with the actual practice in the counties of New Jersey with respect to the need for transportation. The reader, no doubt, will remember a statement in the last chapter to the effect that it was not desirable or possible to secure a criterion to correlate perfectly with present practice. To do so would be merely to duplicate present practice. In accepting this criterion which correlates .81 with present practice it becomes necessary to show that the deviations from this criterion are caused by factors that should not be accepted in a program of equalization. This is the task for the following chapter.

[2] Derived from Yule, *Theory of Statistics*, p. 204.

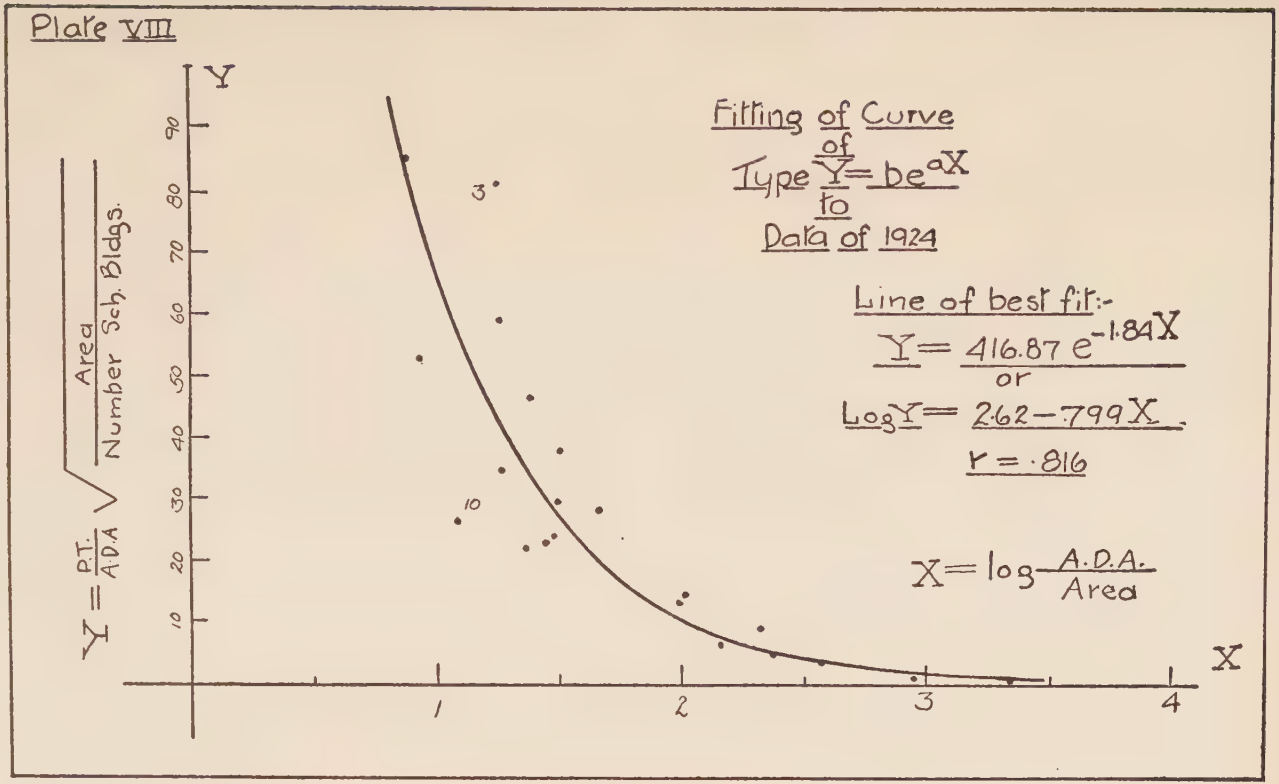
Plate VIII
Y
X
$Y = \frac{P.T.}{A.D.A} \sqrt{\frac{Area}{Number\ Sch.\ Bldgs.}}$
10
20
30
40
50
60
70
80
90
1
2
3
4
Fitting of Curve of Type $Y = be^{aX}$ to Data of 1924
Line of best fit:-
$Y = 416.87\, e^{-1.84X}$
or
$\log Y = 2.62 - .799X$
$r = .816$
$X = \log \frac{A.D.A.}{Area}$

PLATE VIII

SUMMARY

1. The problem of this chapter was to select an empirical formula to show properly the functional relation between the two variables selected in Chapter I.

2. Three mathematical expressions were tried and rejected.

3. The following mathematical expression was selected to describe the functional relation between the density of school population and transportation need:

$$\tilde{Y} = 416.87e^{-1.84X}$$

or

$$\log \tilde{Y} = 2.62 - .799X$$

4. By use of this formula it is possible to determine the transportation need as a function of the density of school population.

5. It is assumed that the value of the transportation need, given by the equation of selected line of best fit, represents a sort of average for counties of like densities, and may properly be taken as a measure of a minimum program in transportation.

CHAPTER IV

STUDY OF FACTORS CAUSING DEPARTURE FROM CURVE

In Chapter III an empirical formula was derived which expressed the functional relationship between the log of density of school population and the transportation need. Prediction based on this formula yielded a correlation of .816 with actual practice in the State of New Jersey. Although it is not necessary or desirable to have predicted transportation need correlate perfectly with practice, it is necessary to determine, if possible, the causes of variation from the curve. Such an examination may tend further to validate the use of this formula in the solution of the problem of equalizing the burden of transportation.

If the distribution of the variables being studied were rectilinear instead of curvilinear, there would be an excellent opportunity to use the method of multiple and partial correlations in determining the influence of the various factors involved. A method may be used, however, that is somewhat analogous to the above method, and it will give a better understanding of the full implications of the formula derived in the preceding chapter.

A measure of the departure from the curve may be defined as follows:

$$\text{Departure} = Y - \tilde{Y}$$

where Y equals the transportation load as defined in Chapter III and $\tilde{Y}$ equals the equation value of Y as found by substituting X in the equation derived in the same chapter. If $\tilde{Y}$ is thought of as the most probable value of the transportation need, and Y as the observed value, then the difference $Y - \tilde{Y}$ may be called a residual. The departure or residual for each county is found by substituting the value X in the derived equation, solving for $\tilde{Y}$ and subtracting this value from Y. The results appear in the last column in Table III.

The problem resolves itself into finding what factors correlate

with the residual. If such factors can be found, they will be factors acting on the transportation need and independent of the density of school population, for by taking the residuals the influence of density has been cancelled out. The residual in each case is that part of the transportation need that is independent of density. If it can be shown that the factors acting on the transportation need, independent of our criterion, are such as reflect local policies with respect to school administration, then it can be concluded that such factors should not be included in the minimum program of transportation to be equalized. The minimum program must necessarily be the average of conditions as they exist for school units of equal densities, and it can not represent the extremes resulting from full expression of local desires with respect to educational opportunities.

In the search for such factors that may influence the transportation need of a county, there is one factor which immediately presents itself; that is, the factor of consolidation of schools. It is quite apparent that consolidation means transportation, and that a large number of one-room schools means the contrary, relatively less need for transportation. The tendency for counties to form large school units will be shown in the types of school buildings in the county and the number of each, respectively. That is to say, it is necessary to know how many one-room school buildings there are in a county, how many two-room buildings, and so on to the largest unit reported. These data were available for the school year of 1924 and they are presented in Table III. There are also shown in Table III (in the last column) the residuals for each county as they were calculated from empirical formula derived in the last chapter.

Since the problem is to discover the causes of departures from the curve, it may be helpful to compare the data from the counties having the greatest departure from the curve in positive and negative values. A glance at Plate VIII will show the two counties, marked 3 and 10, that have the greatest departure from the selected curve with densities almost equal in value. In fact, if these two dots in Plate VIII were covered with small pieces of paper, it would be seen that the curve is almost an ideal fit to the remaining data. The counties numbered 3 and 10 are Burlington and Hunterdon, respectively. Turning to Table III it is possible to find the numerical value for the departures and to discover also other characteristics of these two counties. In the column

TABLE III

RELATION OF TYPES OF SCHOOL BUILDINGS AND RESIDUALS,[3] NEW JERSEY, 1924

County	One-room[1] Rural	%	Two-room[2] Rural	%	One-room	%	Two-room	%	Three-room	%	Four-room	%	Five-room or More	%	Total	$Y - \tilde{Y}$[3]
Atlantic	21	23.6	14	15.7	28	31.5	16	18.0	5	5.62	6	6.7	34	38.2	89	− 4.61
Bergen	7	3.9	3	1.6	18	10.1	14	7.8	4	2.25	21	11.8	121	67.9	178	2.94
Burlington	10	10.2	16	16.3	20	20.4	23	23.5	13	13.30	11	11.2	31	31.6	98	38.11
Camden	16	10.3	6	3.8	25	16.1	24	15.5	4	2.58	38	24.5	64	41.3	155	− 1.74
Cape May	13	24.2	14	25.9	15	27.8	20	37.0	2	3.70	3	5.5	14	25.9	54	− 8.12
Cumberland	21	23.9	12	13.6	28	31.8	18	20.5	4	4.54	13	14.8	25	28.4	88	12.77
Essex	2	1.0	0	0.0	8	4.1	13	6.6	1	.51	16	8.1	158	80.6	196	− .49
Gloucester	14	15.9	16	18.2	24	27.3	21	23.9	7	7.95	19	21.6	17	19.3	88	11.83
Hudson	0	0.0	0	.0	3	2.5	0	.0	0	.00	2	1.7	112	95.7	117	− .16
Hunterdon	61	70.1	10	11.5	62	71.3	10	11.5	2	2.29	4	4.5	9	10.3	87	−32.88
Mercer	7	7.9	5	5.6	11	12.5	7	7.9	2	2.26	16	18.2	52	59.1	88	4.35
Middlesex	22	17.5	7	5.5	28	22.2	16	12.7	5	3.96	15	11.9	62	49.2	126	2.78
Monmouth	35	24.1	19	13.1	45	31.1	32	22.1	4	2.76	16	11.0	48	33.1	145	8.11
Morris	27	21.4	23	18.3	43	34.1	28	22.2	0	.00	23	18.2	32	25.4	126	2.08
Ocean	23	38.9	13	22.0	27	45.8	15	25.4	2	3.39	3	5.0	12	20.3	59	1.07
Passaic	7	7.1	9	9.1	10	10.2	13	13.3	3	3.06	1	1.0	71	72.4	98	− .67
Salem	22	33.8	5	7.6	33	50.8	9	13.8	3	4.62	7	10.8	13	20.0	65	16.92
Somerset	35	45.5	16	20.8	35	45.5	16	20.8	1	1.29	8	10.4	17	22.1	77	− 5.54
Sussex	47	62.7	5	6.6	54	72.0	7	9.3	3	4.00	2	2.6	9	12.0	75	−22.95
Union	2	1.8	1	.9	10	9.0	5	4.5	0	.00	8	7.2	88	79.3	111	− .15
Warren	11	45.5	12	13.3	52	57.7	15	16.7	2	2.22	6	6.6	15	16.7	90	−12.52
Total	433		206		579		322		67		238		1004		2210	

[1] Estimated from number of one-room rural school teachers in New Jersey School Report, 1924.

[2] Estimated from number of two-room rural school teachers in New Jersey School Report, 1924.

"A rural school is one located either in the open country or in the village, the majority of whose pupils are the children of farmers and others living in the country." *Ibid.*

[3] Departure from curve for 1924.

showing the percentage of one-room school buildings in each county, it can be seen that Burlington has 20.4 per cent of its school buildings of the one-room type, while Hunterdon has 71.3 per cent of its buildings in this class. The same condition for these counties is apparent with respect to one-room rural school buildings. These figures seem to say that the program of education in Burlington County with only 20.4 per cent one-room buildings must be richer in opportunity than that of Hunterdon County, where 71.3 per cent of the schools are of the meager one-room type. It is also safe to infer that the program of transportation must be far greater in Burlington than in Hunterdon County. There is then in these two counties a wide divergence in the need for transportation, and it may be concluded that, for these counties, the departure from curve is closely related to the type of school buildings, and hence to the richness of educational opportunity.

The situation in these two counties seems to point to a definite relation between the type of school organization and departure from the curve. The method of correlation may be used to sum up the whole situation and avoid a lengthy analysis of Table III with respect to all counties. In each case the residuals have been correlated against the percentage of different types of buildings. For example, there is a correlation of $-.52$ between the residuals and the percentage of one-room rural schools. This correlation is in harmony with sound logic, for one would expect those counties to have the largest percentage of one-room rural schools that had those residuals least in value.[1] It means graphically that those counties having a large percentage of one-room rural schools would have a small transportation need and hence would fall below the curve in Plate VIII.

The entire situation for the different types of buildings is shown in Plate IX, Fig. 1. Plate IX reveals a striking characteristic in the distribution of these correlations. The distribution is almost rhythmical and similar to the familiar sine curve. The greatest negative value is $-.52$ for one-room rural schools; it drops to practically zero for two-room rural schools, rises to the greatest positive value of .54 for three-room buildings, and again falls quite low, approaching zero for buildings of five rooms or more.

The conclusions to be reached from the foregoing are as follows: (1) one-room school buildings are associated negatively with

[1] The reader should bear in mind that residuals may be either positive or negative in value.

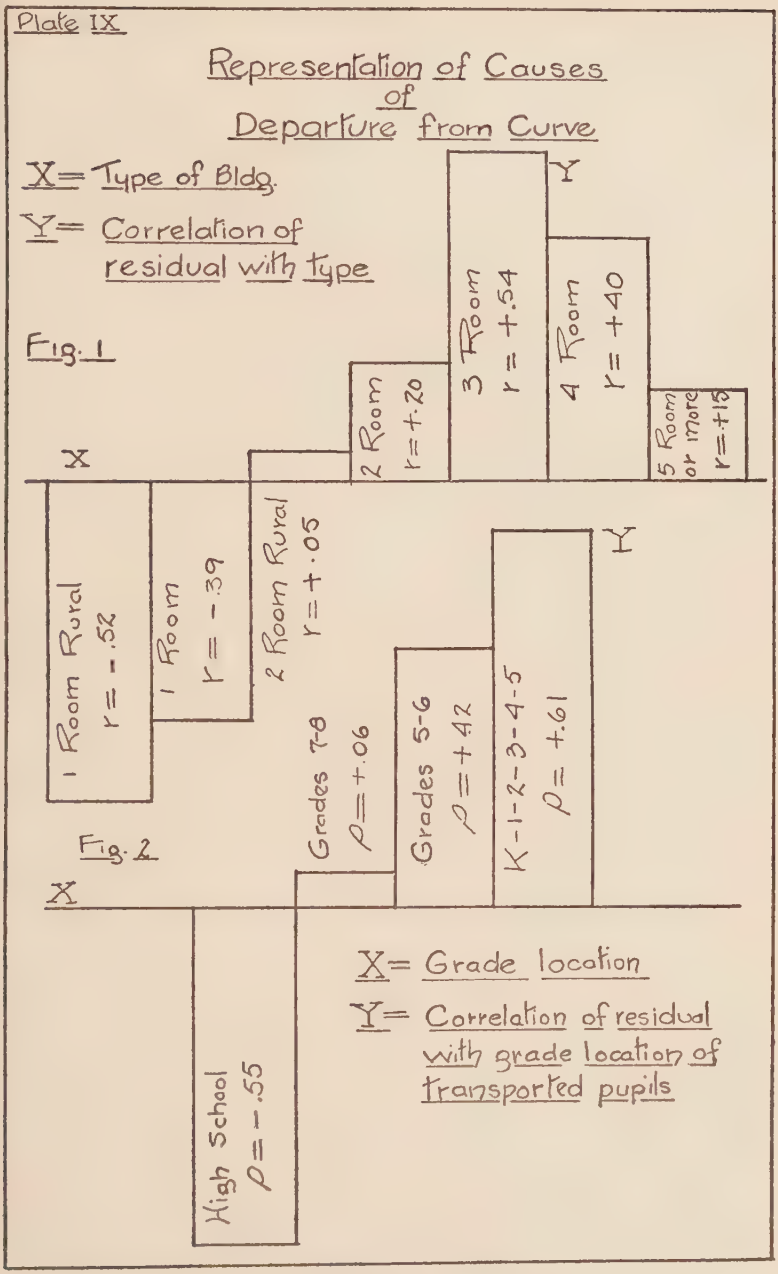
Plate IX
Representation of Causes of Departure from Curve
X= Type of Bldg.
Y= Correlation of residual with type
Fig. 1
X
Y
1 Room Rural r = -.52
1 Room r = -.39
2 Room Rural r = +.05
2 Room r = +.20
3 Room r = +.54
4 Room r = +.40
5 Room or more r = +.15
Fig. 2
X
Y
Grades 7-8 ρ = +.06
Grades 5-6 ρ = +.42
K-1-2-3-4-5 ρ = +.61
High School ρ = -.55
X= Grade location
Y= Correlation of residual with grade location of transported pupils

PLATE IX

departure from the curve; (2) two-room rural schools have no relation to the departures from the curve; (3) three- and four-room school buildings are associated positively with departures from the curve; (4) buildings of five rooms or more tend to have little relation to departures from the curve. Since the departures from the curve; or the residuals, represent that part of the transportation need independent of density of school population, it may be concluded that variability in transportation need for places of like density is caused by local policy with respect to type of school buildings, and, hence, the educational program.

In addition to the types of school buildings in the various counties, there is available for study another factor which may have a strong influence on the departures from our selected curve. From the types of buildings found, conclusions were drawn with respect to the residuals and transportation need. It is now possible to get a more direct measure of local policies with respect to transportation. This will in turn make it possible to study relation between residuals and factors causing them.

In Table IV is presented a statement of the grade location of all transported pupils in the State of New Jersey for the year 1926. These pupils are grouped into four large classes: (1) kindergarten and first four grades, (2) fifth and sixth grades, (3) seventh and eighth grades, and (4) high school. If a county has an extensive program of transportation, one would expect to find a large number of lower grade children being transported to school. If the county is merely following the traditional procedure, one would expect to find that most of the pupils being transported are high school pupils. If a county is transporting a great many children of the lower grades, that means central schools. This would cause a high positive departure from the curve, and, if departures were correlated against the various percentages of transported children found in K–1–2–3–4 grades in school, a high positive correlation should result. It is also possible to say that the results of the foregoing paragraphs should parallel the results that are found here, for they are only another method of saying the same thing. With these questions in mind Table IV is presented for examination.

It will be interesting to note the position of those two counties which were discussed previously, namely, Burlington and Hunterdon. The reader will find that Burlington ranks third in the percentage of transported pupils that are in K–1–2–3–4

TABLE IV

RELATION BETWEEN RESIDUALS AND GRADE LOCATION OF TRANSPORTED PUPILS FOR 1925–1926

County	Total Pupils Transported	K-1-2-3-4	%	5-6	%	7-8	%	High School	%	Residual [1]
Atlantic	1,211	343	28.3	267	22.0	176	14.52	425	35.0	– 4.61
Bergen	4,719	979	20.7	591	12.5	496	10.5	2,653	56.1	2.94
Burlington	4,340	1,653	38.0	1,005	23.1	679	15.6	1,003	23.1	38.11
Camden	2,245	333	14.8	438	19.5	381	16.92	1,093	48.6	– 1.74
Cumberland	1,992	750	37.6	385	19.2	301	15.1	556	27.9	– 8.12
Cape May	672					349	51.9	323	48.06	12.77
Essex	1,147	347	30.2	163	14.21	233	20.3	404	35.1	– .49
Gloucester	2,416	869	35.9	412	17.0	297	12.29	838	34.68	11.83
Hudson	1,006	165	16.4	231	22.9	244	24.2	366	36.3	– .16
Hunterdon	847	148	17.4	83	9.79	44	5.19	572	67.5	–32.88
Middlesex	2,578	725	28.1	492	19.0	437	16.95	924	35.8	4.35
Mercer	2,050	749	36.5	292	14.24	299	14.58	710	34.63	2.78
Monmouth	2,941	770	26.1	280	9.52	482	16.3	1,409	47.9	8.11
Morris	2,250	669	29.7	396	17.6	320	14.2	865	38.4	2.08
Ocean	1,346	581	43.0	189	14.0	183	13.5	393	29.1	1.07
Passaic	1,754	521	29.6	233	13.28	159	9.06	841	48.00	– .67
Somerset	1,161	340	29.20	152	13.0	142	12.23	527	45.3	16.92
Salem	1,524	592	38.8	259	16.9	168	11.0	505	33.1	– 5.54
Sussex	893	261	29.22	90	10.0	113	12.6	429	48.04	–22.95
Union	976	330	33.8	101	10.34	92	9.41	453	36.4	– .15
Warren	647	145	22.4	67	10.35	71	10.9	364	56.2	–12.52

[1] Departure from curve for 1924.

grades, and Hunterdon ranks eighteenth. If all the counties of this group are numbered in this trait and in order of departure from curve, and the coefficient of correlation by the rank method is calculated, it is found to be +.61. This verifies our previous assumptions and shows conclusively that departures from curve are positively related to local policy with respect to central schools and transportation. The same procedure is followed for pupils located in the other divisions and the correlations are for 5–6 grades, 7–8 grades, and high school, +.42, +.068, and −.55, respectively. These results are shown graphically in Plate IX.

The conclusions to be reached here are: (1) there is a high positive correlation between departures from curve and percentage of transported children in K–1–2–3–4 grades; (2) there is a fairly significant correlation between departures and percentage found in 5–6 grades; (3) there is no correlation between percentage found in 7–8 grades and residuals; (4) there is a high negative correlation between percentage of transported pupils who are in the high school and residuals. The last is really a complement of the first.

SUMMARY

1. Departures or residuals may be expressed algebraically as follows:

$$\text{Departure} = Y - \tilde{Y},$$

when Y equals observed transportation need and $\tilde{Y}$ equals the equation value.

2. The residual is that part of transportation need that is independent of density.

3. The equalized transportation program can not be one extreme or the other resulting from free expression of local desire with respect to educational opportunities. It must be the average of conditions as they exist for school units of equal densities.

4. Study of the relation between residuals and types of school buildings shows that variability from the curve of transportation need, for places of like density, was caused by local policy with respect to type of school buildings and educational program.

5. Study of the relation between residuals and grade location of transported pupils further validates (4), showing conclusively that departures from curve are associated positively with larger school units and more extensive programs of centralization and transportation.

6. The equation derived in Chapter III may properly be taken as an index of the transportation need in a problem of equalizing a minimum program of transportation.

7. The next problem is to apply this index to the apportioning of state money for transportation for one school year.

CHAPTER V

APPLICATION OF CURVE TO NEW JERSEY COUNTIES

In Chapter III an empirical formula was derived to measure transportation need and to furnish a basis for an equalization of transportation costs in New Jersey. There are two important factors in a program of equalizing the burden of transportation: (1) the measurement of the transportation need, and (2) the translation of such a measure into dollars and cents in the determination of the cost of the minimum program of transportation to be equalized by the state. The writer believes the first of these factors has been satisfactorily handled by the derivation of the empirical formula. The second is specifically the problem of this chapter. The solution, therefore, will consist of two parts: (1) measuring the transportation need of the counties in terms of the minimum program, and (2) apportioning state aid to equalize the transportation costs on the basis of the state's minimum program.

It might be well at this point to discuss briefly the use of the empirical equation in the measurement of transportation need. This equation involves two variables X and Y. The X value is the log of the density of school population, the $\tilde{Y}$ value is the measure of transportation need. It is really a ratio of which the denominator is the average daily attendance and the numerator the measure of units of need which we desire to know.

The equation showing the functional relation between these two variables may be written in two forms as follows:

$$\tilde{Y} = 416.87e^{-1.84X}$$
$$\log \tilde{Y} = 2.62 - .799X$$

The second form is much easier to handle and it will be used throughout this chapter. Its use demands a knowledge of the use of logarithms. It can be readily seen that when X is substituted in this equation $\tilde{Y}$ can at once be determined. The $\tilde{Y}$ value thus determined is a ratio, the ratio of units of transporta-

tion need of a given county to the average daily attendance. Knowing $\tilde{Y}$ and the average daily attendance, one finds the units of transportation need for the given county by multiplying the two values, $\tilde{Y}$ and A.D.A. The $\tilde{Y}$ used here is quite analogous to the b value found in the method used by Mort. Heretofore the $\tilde{Y}$ value has been expressed as a per cent, but hereafter it will be turned into a decimal fraction to facilitate multiplication. It may be helpful to work through one example for the reader. The data are taken from Atlantic County for 1924:[1]

$$X = \log \frac{\text{A.D.A.}}{\text{area}} = \log \frac{16771}{569} = 1.46945$$

$$\begin{aligned} \log \tilde{Y} &= 2.62 - .799X \\ &= 2.62 - .799 \times 1.46945 \\ &= 2.62 - 1.1740 \\ \log \tilde{Y} &= 1.4460 \\ \tilde{Y} &= 27.93\% \text{ or } .2793 \end{aligned}$$

If the $\tilde{Y}$ value for Atlantic County is .2793, then the transportation units for that county will be equal to A.D.A. times .2793 or 16771 times .2793, which is 4684.14. In every case the $\tilde{Y}$ value is found by substituting in the empirical equation. This $\tilde{Y}$ value when multiplied by the A.D.A. yields the units of transportation need for that county. The results for all counties are shown in Table V.[2]

The measurement of the transportation need for all the counties of New Jersey, as presented in Table V, is the first step in the application of the derived curve to New Jersey counties. The units of transportation need for each county are now known, and the next step will be to determine the cost of the program of transportation to be equalized. It may be said, in other words, that the problem is to transmute the need into dollars. The same problem is found wherever there is the necessity of arriving at the cost of a minimum program. One much used method of arriving at the cost of the satisfactory minimum program is the state-wide expenditure per unit of educational need. This method was used by the New York State Finance Inquiry[3] and by Singleton.[4] Mort introduced a more refined method when he

[1] Wherever it was possible the writer has relied on the use of logarithms and the slide rule for calculation.

[2] These results were obtained by use of the derived equation. The writer presents a simpler method at the close of this chapter.

[3] Strayer, G. D. and Haig, R. M. *Financing Education in the State of New York*, p. 12.

[4] Singleton, G. G. *State Responsibility for Support of Education in Georgia*, p. 20.

TABLE V

Application of Transportation Index for 1924

County	A.D.A.	Area in Sq. Miles	$\frac{\text{A.D.A.}}{\text{Area}}$	$\frac{\log \text{A.D.A.}}{\text{Area}}$	$\tilde{Y}$ Value	Transportation Need [1]
Atlantic	16,771	569	29.47	1.46945	.2793	4,684.14
Bergen	49,097	237	207.16	2.31631	.0589	2,891.81
Burlington	14,086	815	17.28	1.23763	.4278	6,025.99
Camden	31,384	222	141.37	2.15036	.0798	2,504.44
Cape May	4,654	265	17.56	1.24458	.4223	1,965.38
Cumberland	11,597	500	23.19	1.36538	.3382	3,922.11
Essex	113,745	127	895.62	2.95212	.0191	2,172.53
Gloucester	10,499	332	31.62	1.50001	.2641	2,759.58
Hudson	92,444	43	2149.86	3.33241	.0091	841.24
Hunterdon	5,123	437	11.72	1.06904	.5834	2,988.76
Mercer	23,734	226	105.01	2.02126	.1012	2,401.88
Middlesex	30,621	312	98.14	1.99187	.1070	3,276.45
Monmouth	21,501	479	44.88	1.65212	.1995	4,289.45
Morris	14,825	475	31.21	1.49430	.2268	3,955.31
Ocean	4,787	637	7.51	.87592	.8325	3,985.17
Passaic	45,142	196	230.31	2.36232	.0545	2,460.24
Salem	5,984	343	17.44	1.24170	.4246	2,540.81
Somerset	8,677	305	28.43	1.45382	.2874	2,493.77
Sussex	4,508	529	8.52	.93052	.7526	3,412.72
Union	37,754	103	366.53	2.56412	.0356	1,344.04
Warren	8,186	362	22.61	1.35436	.3451	2,824.99
Total						63,740.81

[1] Transportation Need = A.D.A. $\times$ $\tilde{Y}$ value.

found the central tendency of the unit expenditure in all the communities of average wealth per unit of educational need; this central tendency was then taken as the proposed unit cost of the minimum program.[5] He has this to say of both methods:

> Inasmuch as the central tendency of expenditure in the state as a whole may be expected to approximate the expenditure in communities of average wealth, such measures should be reasonably acceptable.

Following this line of attack the first problem that presents itself is to discover the cost of one unit of the minimum transportation program. If the cost of one unit and the number of units in a given county are known, the product of these two items should give the cost of the minimum program of transportation which is to be borne by the state. Material for this analysis is presented in Table VI. This table shows that the cost per unit of need varies from $11.75 in Atlantic County to $32.82 in Hudson County. The variation in cost per pupil transported is indeed great, as it includes Union County at $15.81 and Hunterdon County at $88.61. The median cost per unit of need is $19.65; the average unit cost is $20.43. These last two figures, $19.65 and $20.43, are sound reason for taking $20 as the cost of one unit of transportation need. The cost of the minimum program in dollars may, therefore, be found for each county by multiplying the need of that county by 20. These results are shown in Table VI.

If the state is to support completely the $20 program of transportation, it must prepare to spend the sum of $1,274,816.20 for transportation. If the cost of the minimum program is equalized by a combination of state and local support, the cost to the state of such a program may be much less. The combined state and local support plan for the equalization of the cost of a minimum program is called the small fund plan.[6]

It is at this point that a correction should be used but, unfortunately, that correction is not available at the present time. It was pointed out in the first chapter that the pioneer study in the measurement of educational need by Mort needed to be corrected in terms of the cost of living. The same correction may be needed here, for the cost of maintenance and operation of

[5] Mort, Paul R. *State Support for Public Schools*, p. 20.

[6] For a discussion of the small fund plan, see Mort, Paul R. *State Support for Public Schools*, p. 25.

TABLE VI

State Aid Based on Transportation Index for 1924

County	Total Cost of Transportation	Cost per Pupil Transported	Cost per Unit of Need	Estimated State Aid [1]	Cost of $20 Program Need × 20	Cost of $15 Program Need × 15	Present [2] Local Contribution	Proposed Local Contribution [3]
Atlantic	$ 55,047.63	$34.84	$11.75	$ 41,285.72	$ 93,682.80	$ 70,262.10	$13,761.91	none
Bergen	75,235.33	19.84	26.01	56,426.50	57,836.20	43,377.15	18,808.83	$17,399.13
Burlington	108,207.88	27.19	17.96	81,155.91	120,519.80	90,389.85	27,051.97	none
Camden	60,355.68	36.85	24.10	45,266.76	50,088.80	37,566.60	15,088.92	10,266.88
Cape May	27,753.28	38.33	14.12	20,814.96	39,307.60	29,480.70	6,938.32	none
Cumberland	67,521.88	29.58	17.25	50,641.41	78,442.20	58,831.65	16,880.47	none
Essex	58,102.63	30.63	26.74	43,576.97	43,450.60	32,587.95	14,525.66	14,652.03
Gloucester	69,603.68	33.69	25.21	52,202.76	55,191.60	41,393.70	17,400.92	14,412.08
Hudson	27,609.36	23.99	32.82	20,707.02	16,824.80	12,618.60	6,902.34	10.784.56
Hunterdon	53,518.81	88.61	17.91	40,139.11	59,775.20	44,831.40	13,379.70	none
Mercer	75,205.44	35.04	31.31	56,404.08	48,037.60	36,028.20	18,801.36	27,167.84
Middlesex	77,967.27	29.67	23.80	58,475.45	65,529.00	49,146.75	19,491.82	12,438.27
Monmouth	123,516.83	37.08	28.80	92,637.62	85,789.00	64,341.75	30,879.21	37,727.83
Morris	92,431.93	40.50	23.37	69,323.95	79,106.20	59,329.65	23,107.98	none
Ocean	50,687.40	40.84	12.72	38,015.55	79,703.40	59,777.55	12,671.85	none
Passaic	42,290.90	27.88	17.19	31,718.18	49,204.80	36,903.60	10,572.72	none
Salem	48,579.94	31.34	19.12	36,434.96	50,816.20	38,112.15	12,144.98	none
Somerset	49,011.90	48.05	19.65	36,758.93	49,875.40	37,406.55	12,252.97	none
Sussex	68,109.63	77.74	19.96	51,082.22	68,254.40	51,190.80	17,027.41	none
Union	23,453.44	15.81	17.45	17,590.08	26,880.80	20,160.60	5,863.36	none
Warren	47,697.02	52.47	16.88	35,772.76	56,499.80	42,374.85	11,924.26	none
Total	$1,301,907.86	$33.63	$20.43	$976,430.90	$1,274,816.20	$956,112.15		

Median Unit Cost = $19.65

[1] Calculated at 75% cost.
[2] Total cost of transportation less estimated State Aid.
[3] Local contribution to supplement proposed State Aid based on equalization program.

transportation lines over a wide area is affected by variations in the cost of living. The cost of the minimum program of transportation should, therefore, be adjusted in terms of the cost of living in the counties of New Jersey. If an index of such costs were available, then the measurement of transportation need and the apportionment of state aid on such measurements would be on a very sound and equitable basis.

The procedure thus far can be briefly summarized as follows:

(1) By use of the empirical equation determine the number of transportation units in a given county.

(2) Multiply the number of units by the unit cost, $20; the result will be the state aid to equalize the burden of transportation.

(3) Applications of (1) and (2) are shown in Tables V and VI.

There is one very important adjustment that must be made, however. Examination of Table VI will show that the costs of transportation in a few counties are a little less than the proposed state aid and the contrary is observed in other counties. The method proposed here is to apportion full costs where costs are below the state aid proposed. Where costs are above the state aid found by this method, the writer would advocate state aid up to the equalization level and complete county support for the remainder of the costs.

The reader will recall that in Chapter I it was stated that the county superintendents apportion state moneys on the basis of teachers, tuition, and transportation. The remainder, if there is any, is apportioned on the basis of attendance. In Chapter II it was shown that the county superintendent has full supervision and right of approval or disapproval of all transportation and contracts for the same. One great source of state funds is the state school tax, 10 per cent of which is put in the state reserve fund and the remainder sent back to the counties from whence it came for distribution within the county.

With these points in mind, it appears quite justifiable to place transportation wholly on a county basis, to equalize the burden among the counties by use of the index here developed, to place the burden of supplementing the state's contribution on the whole county, and, finally, to lodge complete supervision of transportation and apportioning of state and county funds received for this purpose in the county superintendent. The state fund for equalizing the burden of transportation should come from a source similar to the state-wide uniform school tax. The county con-

tribution, if one is needed, should come from a fund similar in nature to the 90 per cent of state school tax which is raised locally. By this method there would be state support of the minimum program and county support of the supplementary funds necessary. Such a plan as that proposed here would be a great advance in the equalization of educational opportunity in the State of New Jersey.

The index of transportation developed here yields a measure in terms of weighted pupils. This measure, when properly weighted, may be combined with the measure of educational need which is also expressed in weighted pupils. Since the educational need is the sum of all the factors that affect the cost of the minimum program, the weights should be in proportion to the unit cost of each factor. If the cost per unit of educational need is $60 and the cost per unit of transportation need is $20, the total educational need is the number of weighted pupils found by measurement of educational need, plus one-third of the weighted pupils found by use of the transportation index. The resulting figure, the total educational need, can be used (in studying the ability of various communities to support education) to determine the taxable wealth or income per unit of total educational need.

Earlier in this chapter it was stated that a simple method for using the empirical equation derived in Chapter III would be presented and applied in this chapter.

The writer is fully aware of the fact that an exponential or logarithmic equation presents a terrifying aspect to most people. If the index of transportation developed here is to be of practical value, it must be shorn of all relationship to abstract mathematics and tied up to everyday experiences. With this in mind, a table has been prepared based on the fundamental equation derived in these pages. This table appears as Table VII and its application is simple enough to be readily understood by the careful student. The results from it give the same result as the theoretical curve. To use this table one needs to know only the density of school population for a given county; the table yields the $\tilde{Y}$ value to two places, and the third place, if desired, may be found by interpolation. The use of the table demands no knowledge of the mathematical or statistical theory underlying its derivation. The construction of such a table is fundamental to the development of an index of transportation which will be of practical use. If this study were to close with the presentation of an exponential or

TABLE VII

Table Showing $\bar{Y}$ Value for Density of School Population

	0	1	2	3	4	5	6	7	8	9
.00	000000	33,936.204	14,252.951	8,580.682	5,986.133	4,527.618	3,603.781	2,971.435	2,514.131	2,169.554
.0	000000	1,901.510	798.620	480.787	335.415	253.684	201.869	166.496	140.872	121.564
.1	106.545	94.565	84.808	76.724	69.927	64.143	59.165	57.842	51.056	47.716
.2	44.747	42.097	39.717	37.567	35.619	33.844	32.224	30.737	29.369	28.107
.3	26.939	25.856	24.849	23.910	23.033	22.212	21.443	20.720	20.040	19.399
.4	18.793	18.222	17.681	17.167	16.681	16.218	15.778	15.359	14.960	14.578
.5	14.214	13.866	13.534	13.215	12.912	12.616	12.334	12.065	11.805	11.555
.6	11.314	11.083	10.859	10.644	10.436	10.236	10.042	9.855	9.674	9.499
.7	9.329	9.165	9.006	8.852	8.702	8.557	8.417	8.280	8.148	8.018
.8	7.893	7.771	7.653	7.538	7.426	7.317	7.210	7.107	7.006	6.907
.9	6.811	6.718	6.627	6.538	6.449	6.366	6.283	6.202	6.122	6.045
1.0	5.970	5.896	5.824	5.753	5.684	5.616	5.550	5.485	5.422	5.359

Use of table to determine $\bar{Y}$

(1) Density for Morris County = 31.21; locate this value in table, note reading .26

(2) Find difference between two nearest table values; $32.224 - 30.737 = 1.487$

(3) Find difference between larger table value and given value; $32.224 - 31.210 = 1.014$

(4) Determine decimal to be added to .26; $.01 \times \frac{1.014}{1.487} = .0068$

(5) $\bar{Y}$ value = .26 + .0068 = .2668 ans.

Transportation Need = A.D.A. $\times \bar{Y}$

$= 14{,}825 \times .2668 = 3955.31$

This is exactly equal to equation value.

logarithmic equation, it is to be feared that the whole process would be labor lost, for it would have little influence on actual practice. It is to be hoped, however, that the table lifts the solution out of the field of pure or abstract mathematics and places it in the hands of the worker in the field.

SUMMARY

1. The empirical formula developed in Chapter III was used to measure the transportation need and to furnish a basis for the equalization of the burden of transportation in New Jersey counties.

2. The fundamental equation can be written in two forms:

$$\tilde{Y} = 416.87e^{-1.84X}$$

and

$$\log \tilde{Y} = 2.62 - .799X$$

3. The logarithmic form is used and the $\tilde{Y}$ value is calculated for all counties. The $\tilde{Y}$ value when multiplied by the A.D.A. yields in turn the number of units of transportation need.

4. An analysis of costs of transportation on the basis of unit of need gives $20 as an acceptable unit cost for determining the cost of the minimum program of transportation to be borne by the state.

5. The product of unit cost and number of units in a given county gives the total state aid based on an equalization program.

6. Two plans are presented, based on unit costs of $20 and $15. The first is recommended as the more equitable plan.

7. A correction of cost of minimum program in terms of cost of living in various counties is suggested as a means of further validating proposed equalization of costs.

8. It is recommended that the state pay or equalize all transportation costs up to the level of the minimum program. Counties that spend more do so out of local funds. Counties that spend less than is allowed by the state minimum program receive an amount equal to actual expenditures for transportation.

9. It is proposed to place all authority for approval of contracts for transportation and the apportioning of county funds for transportation in the hands of the county superintendents.

10. The state fund for equalizing the transportation costs should be raised by a method that would bear, in an equitable

manner, on all the state. If there were equality in assessments, the present state school tax would meet the situation. The deficit to be met in some counties should be raised locally as a county project. The state, therefore, would equalize the burden of transportation between the counties; and within the counties the additional burden, if any, should be borne by the whole county.

11. A table is presented which is based on the mathematical curve. This table is relatively simple and can be used without any knowledge of its derivation. Such a table should be the outcome of any attempt to solve the problem of equalizing the costs of transportation in a state on a practical basis.

CHAPTER VI

SUMMARY

A modern program of state aid based on the principle of equalization of educational opportunity demands an index of transportation which will enable the state to distribute the burden of transportation costs in an equitable manner. The data for this study were secured in the State of New Jersey, but it was the intent to discover a technique which, with proper adjustments, would be applicable to any state. It should be recognized at the outset that local practice and glaring inequalities in education make it very difficult to discover the law which is operating to produce a transportation need in a given school unit.

The result of this study can be set forth in a mathematical expression which is proposed as an index of transportation. This expression involves two variables X and Y, which may be defined as follows:

$$X = \log\left(\frac{\text{A.D.A.}}{\text{Area}}\right)$$

$$Y = \frac{\text{P.T.}\sqrt{\dfrac{\text{Area}}{\text{Number of School Buildings}}}}{\text{A.D.A.}}$$

X has been defined as the criterion and $\tilde{Y}$ as the measure of transportation need in a given county. The reliability and validity of X as a criterion have been checked for a period of five years. The mathematical expression is the empirical formula representing the line of best fit to these variables for the year 1924. It is proposed to take the equation value, or the $\tilde{Y}$ value, as a measure of the minimum program of transportation for which the state is responsible. Deviations from this minimum program are shown to be the result of local policy. The mathematical expression is put in simple form by constructing a table based on it. The use of this table does not require an understanding of its derivation. The table makes it possible for anyone to utilize the results of mathematical and statistical research.

The use of this technique in the measurement of transportation need has been applied to New Jersey, and a method is shown for determining the cost of the minimum program. A method of state aid is proposed which involves (1) equalization of transportation burden among counties by use of this index, and (2) county support of any program in advance of the minimum program. It would be highly desirable to correct the cost of the minimum program in terms of the cost of living.

The technique which is presented in this study may, with proper adjustments, be applied to any state. It is not claimed that the table here presented is applicable to other states. It is believed that such a table must be developed for the state in question. There is every reason to believe, however, that similar tables would result and such studies for different states would add considerably to the desired information on the subject of transportation need. One who is interested in such a study needs only the following measures for the school units under investigation: (1) average daily attendance, (2) number of pupils transported to school, (3) area of school unit, and (4) number of school buildings of the unit.

The index of transportation here developed yields a measure of need in terms of weighted pupils. This measure may properly be integrated and combined with the pioneer measurement of educational need. This summation will enable one to rate more equitably unlike communities, e.g., urban and rural, with respect to total educational need. It will also serve to determine in a just manner the taxable wealth or income back of each weighted pupil when one wishes to compare the ability of various communities to support education. It is hoped that this index will help to define more precisely the scope of the minimum program, and assist a state to measure in objective terms its progress toward a great American ideal, "Equalization of Educational Opportunity."

APPENDIX

TABLE VIII

Derivation of Variables X and Y for 1921

County	A.D.A.[1]	Number[1] Pupils Transp.	% A.D.A. Transp.	Area[2]	$\frac{\text{A.D.A.}}{\text{Area}}$	Number[1] Bldgs.	$\sqrt{\frac{\text{Area}}{\text{Bldgs.}}}$	$\frac{\%\text{ A.D.A. Transp.}}{\sqrt{\frac{\text{Area}}{\text{Bldgs.}}}}$	$\frac{\text{Log A.D.A.}}{\text{Area}}$
Atlantic	14,255	992	6.961	569	25.04	91	2.500	17.40	1.39870
Bergen	40,387	2,197	5.440	237	170.30	167	1.191	6.48	2.23142
Burlington	12,252	3,906	31.880	815	15.03	92	2.976	94.90	1.17698
Camden	27,133	1,189	4.382	222	122.20	153	1.204	5.279	2.08710
Cape May	3,636	443	12.190	265	13.72	56	2.175	26.50	1.13737
Cumberland	10,539	1,477	14.020	500	21.06	80	2.500	35.06	1.32346
Essex	101,821	1,227	1.205	127	801.50	184	.830	1.00	2.90395
Gloucester	8,305	1,381	16.63	332	25.01	87	1.953	32.48	1.39820
Hudson	84,976	789	.9285	43	1976.00	119	.601	.55	3.29530
Hunterdon	4,855	605	12.460	437	11.11	82	2.308	28.76	1.04571
Mercer	21,215	1,708	8.052	226	93.85	86	1.619	13.03	1.97243
Middlesex	25,417	1,787	7.032	312	81.44	107	1.707	12.00	1.91085
Monmouth	18,816	2,321	12.330	479	39.26	145	1.817	22.42	1.59405
Morris	12,950	1,410	10.880	475	27.26	114	2.041	22.22	1.43558
Ocean	3,955	1,071	27.070	637	6.208	61	3.231	87.50	.79301
Passaic	40,064	823	2.054	196	204.30	96	1.428	2.935	2.31045
Salem	6,068	1,018	16.77	343	17.69	77	2.110	35.40	1.24776
Somerset	7,518	513	6.824	305	24.64	76	2.003	13.66	1.39180
Sussex	4,167	799	19.17	529	7.877	86	2.480	47.55	.89636
Union	31,385	887	2.826	103	304.6	92	1.058	2.99	2.48381
Warren	7,136	751	10.52	362	19.71	89	2.016	21.22	1.29474
Total	486,850	27,294				2,140			

[1] New Jersey School Report, 1921.
[2] U. S. Census, 1920.

TABLE IX

Derivation of Variables X and Y for 1922

County	A.D.A.[1]	Number[1] Transp. Pupils	% A.D.A. Transp.	Area[2]	$\frac{\text{A.D.A.}}{\text{Area}}$	Number[1] Bldgs.	$\sqrt{\frac{\text{Area}}{\text{Bldgs.}}}$	Y $\frac{\%\text{ A.D.A. Transp.}}{\text{Area}} \times \sqrt{\frac{\text{Area}}{\text{Bldgs.}}}$	X $\text{Log}\frac{\text{A.D.A.}}{\text{Area}}$
Atlantic	15,496	1,115	7.195	569	27.23	90	2.514	18.091	1.43516
Bergen	43,127	2,662	6.172	237	181.97	169	1.184	7.309	2.26000
Burlington	12,974	3,547	27.339	815	15.91	93	2.960	80.933	1.20191
Camden	28,717	1,378	4.798	222	129.35	159	1.181	5.669	2.11179
Cape May	4,124	518	12.292	265	15.90	52	2.252	27.685	1.20144
Cumberland	11,096	1,731	15.600	500	22.192	81	2.484	38.758	1.34620
Essex	107,946	1,626	1.506	127	849.995	189	.819	1.234	2.92939
Gloucester	9,164	1,725	18.823	332	27.602	89	1.931	36.356	1.44095
Hudson	89,377	981	1.097	43	2078.534	116	.608	.668	3.31776
Hunterdon	5,224	497	9.513	437	11.95	88	2.228	21.201	1.07752
Mercer	22,089	1,853	8.388	226	97.73	90	1.584	13.293	1.99007
Middlesex	27,379	2,084	7.611	312	87.75	112	1.669	12.703	1.94327
Monmouth	20,191	2,641	13.080	479	42.15	147	1.805	23.611	1.62482
Morris	13,881	1,931	13.911	475	29.22	128	1.926	26.797	1.46573
Ocean	4,348	1,024	23.550	637	6.82	64	3.154	74.300	.83415
Passaic	42,358	1,314	3.102	196	216.11	95	1.436	4.455	2.33468
Salem	5,997	1,457	24.295	343	17.48	63	2.333	56.689	1.24264
Somerset	7,990	763	9.549	305	26.19	75	2.016	19.257	1.41825
Sussex	4,416	916	20.742	529	83.47	88	2.451	50.858	.92157
Union	34,149	1,156	3.385	103	331.54	96	1.035	3.506	2.52054
Warren	7,773	847	10.896	362	21.47	91	1.994	21.733	1.33188

[1] New Jersey School Report, 1922.
[2] U. S. Census, 1920

TABLE X

Derivation of Variables X and Y for 1925

County	A.D.A.[1]	Number[1] Pupils Transp.	% A.D.A. Transp.	Area[2]	$\frac{\text{A.D.A.}}{\text{Area}}$	Number[1] Bldgs.	$\sqrt{\frac{\text{Area}}{\text{Bldgs.}}}$	% A.D.A. Transp. $\times \sqrt{\frac{\text{Area}}{\text{Bldgs.}}}$	Log $\frac{\text{A.D.A.}}{\text{Area}}$
Atlantic	17,524	1,713	9.777	569	30.79	94	2.460	24.05	1.48842
Bergen	51,858	4,029	7.770	237	218.70	184	1.134	8.819	2.34000
Burlington	14,842	4,683	31 550	815	18.20	95	2.929	92.430	1.26027
Camden	33,194	2,192	6.604	222	149.50	172	1.136	7.503	2.17466
Cape May	4,951	806	16.280	265	18.68	56	2.175	35.410	1.27144
Cumberland	11,782	2,395	20.330	500	23.56	83	2.450	49.90	1.37218
Essex	116,007	2,558	2.205	127	913.40	186	.826	1.82	2.96066
Gloucester	11,208	2,778	24.80	332	33.73	89	1.931	47.90	1.52808
Hudson	93,191	1,001	1.074	43	2167.00	117	.606	.65	3.33590
Hunterdon	5,398	743	13.760	437	12.35	88	2.228	30.67	1.09175
Mercer	24,553	2,382	9.702	226	108.60	85	1.630	15.82	2.03594
Middlesex	32,838	2,446	7.450	312	105.20	126	1.573	11.72	2.02212
Monmouth	21,999	3,300	15.000	479	45.90	146	1.811	27.18	1.66189
Morris	15,173	2,395	15.780	475	31.93	119	1.997	31.54	1.50430
Ocean	4,852	1,437	29.610	637	7.617	58	3.313	98.14	.88179
Passaic	45,924	2,000	4.355	196	234.20	101	1.393	6.068	2.36974
Salem	6,214	1,711	27.470	343	18.11	66	2.279	62.620	1.25808
Somerset	9,327	1,444	15.540	305	30.45	78	1.977	30.740	1.48364
Sussex	4,738	1,024	21.610	529	8.956	74	2.673	57.78	.95214
Union	40,119	1,440	3.590	103	389.4	125	.907	3.258	2.59041
Warren	8,294	965	11.630	362	22.91	83	2.088	24.290	1.36005
Total	573,986	43,442				2225			

[1] New Jersey School Report, 1925.
[2] U.S. Census, 1920.

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